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OF THE

PHYSICAL SOCIETY OF LONDON.

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PROCEEDINGS

OF

THE PHYSICAL SOCIETY

OF LONDON.

March 21, 1874.

I. *On the new Contact Theory of the Galvanic Cell.*

By J. A. Fleming, *B.Sc., F.C.S.*

The contest that has for so long been waged between the supporters of the two theories of the galvanic cell, the *contact* and the *chemical*, can hardly be said to have been brought even now to a decisive issue. For, although the contact theory, as originally proposed by Volta, received a fatal blow when the law of conservation of energy became clearly understood, yet in its place a *new contact theory* has arisen, supported by novel and important experimental evidence, which has again been placed by recent writers on electrical science in formidable opposition to its old rival.

The old contact theory of Volta had its origin in an entire ignorance of the science of energy. It simply referred the current produced through the circuit of a pile to the effect of the metallic contacts, and it ignored the thermal and chemical changes which are also necessarily present; but it had to be finally abandoned when once it became clearly understood that the appearance of a current involved the disappearance of some other energy, actual or potential, as an invariable accompaniment. The new contact theory may be said to have had its source in the discovery of Sir W. Thomson, that there is undoubtedly a difference of potential produced when dissimilar metals are placed in contact—a fact not only abundantly proved by Thomson by direct experimental evidence, but, as he has pointed out, confirmed in a remarkable way by the phenomena of the Peltier effect, which, when interpreted by the dynamical theory of heat, furnish the most reliable measure of its amount. These facts, together with others presently to be referred to, have been made to furnish the key to a fresh explanation of the dynamics of the galvanic cell, which I have ventured to call the *new* contact theory, as opposed to the old or voltaic one.

It is not possible, however, to define in a few words the precise details of the new theory; they can only be arrived at by collecting together the statements as we find them laid down by their authors. The object of the present paper is to draw the attention of those interested in this question to the objections that may be raised against this new contact theory—objections based on facts, some old and some which perhaps may prove new, but all of which alike seem to throw fresh difficulties in the way of this theory, although capable of simple explanation by the old chemical hypothesis. It will be necessary then to review briefly the precise statements of this new contact theory, in order to show exactly what are the points against which objection may be taken. This will be best accomplished by collecting the statements of its principal supporters and arranging together their explanations of the phenomena which arise

(1) when dissimilar metals at the same temperature are placed in contact,

(2) when one insulated metal is placed in a liquid capable of acting chemically upon it,

(3) when two different metals are placed insulated and unconnected in one such liquid,

(4) when the two metals are joined across by a metallic arc or when two or more cells are joined up in series.

1. That the contact of metals is always attended with the production of a difference of potential between them was for a long time denied by ardent supporters of the chemical theory. De la Rive endeavoured to show that the effects observed might be attributed to oxidation; but his experiments are not conclusive;

and to Sir W. Thomson belongs the credit of having established the fact by experiment, irrespective of his theoretical deductions from the facts of thermoelectricity. He thus describes his decisive experiment:—

"A metal bar insulated so as to be movable about an axis perpendicular to the plane of a metal ring, made up half of copper and half of zinc, the two halves being soldered together, turns *from* the zinc *towards* the copper when positively electrified, and *from* the copper *towards* the zinc when negatively electrified"*. The difference of potential he finds to be about ·6 or ·7 of that of a Daniell's cell when the metals are perfectly clean; but by oxidation of the copper it may be made equal to or even greater than that of a Daniell's cell. He has also shown that if zinc and copper cylinders be connected by a wire, the electrometer detects a difference between the potentials of the air in the interior, and, lastly, that if copper filings be allowed to fall from a copper funnel in contact with a vertical zinc cylinder, they convey a negative charge to a receiver placed below. Sir W. Thomson concludes that there is sufficient evidence to show that zinc and copper attract one another chemically at any distance if connected by a fine wire, and that, as Professor Tait remarks, "when *any* two bodies of different kinds are brought into contact, there is a certain amount of exhaustion of the potential energy of chemical affinity between them, and that the equivalent of this is, partly at least, developed in the new potential form of a separation of the so-called electric fluids, one of the bodies receiving a positive, the other a negative change, the quantity depending on the nature and form of the bodies"†.

This is equivalent to saying that at the surface of contact there is perpetually a force tending to separate the two electricities in a direction perpendicular to that surface, while at all points ever so little within it there is no such force.

Professor Maxwell reiterates essentially the same facts. He gives Thomson's proof that the electromotive contact-force at a junction of two metals is represented by PJ, where P= the coefficient of the Peltier effect, or the heat absorbed at the junction due to the passage of a unit of current for a unit of time; and J is Joule's equivalent. He remarks that the electromotive force, as determined by this method experimentally, does not account for the whole electromotive force of a simple couple. This latter is in general far greater than that given by the Peltier effect for the same pair of metals. "Hence the greater part of Volta's force must be sought for, not at the junctions of the two metals, but at one or both of the surfaces which separate the metals from the air or other medium which forms the third element of the circuit"*.

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* Reprint of papers on Electrostatics and Magnetism, p. 316, § 400.

† Thermodynamics, p. 62, § 107.

Professor Jenkin, referring to these experiments of Thomson, adds that "In cases where no known chemical action occurs, as where zinc and copper touch each other, and yet difference of potential is produced, since this involves a redistribution of electricity, a small but definite consumption of energy must then occur; the source of this power cannot yet be said to be known"†.

2. It seems to be universally admitted that when an insulated metal is placed in a liquid capable of acting chemically upon it, a difference of potential is produced between the metal and the liquid, a sudden rise in potential taking place in passing from the metal-surface to the liquid in contact with it, or that the metal becomes negatively and the liquid positively electrified, metals differing in the degree of electrification they can produce with any one electrolyte.

3. But if we ask what are the conditions when two different metals are so immersed, we find the most contradictory statements given. Sir W. Thomson expresses his opinion thus in 1862:—"For nearly two years I have felt quite sure that the proper explanation of voltaic action in the common voltaic arrangement is very near Volta's. I now think it quite certain that two metals dipped in one electrolytic liquid will (when polarization is done away with) reduce *two* dry pieces of the same metals when connected each to each by metallic arcs to the same potential"‡, which seems equivalent to saying that there is no difference of potential produced other than that due to dissimilar contact. Thus also Professor Tait:—"By interposing between two metals which have been electrified by contact a compound liquid or electrolyte, these metals are at once reduced to the same potential—a result which could not have been obtained by

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* Treatise on Electricity and Magnetism, vol. i. p. 302.

† Electricity and Magnetism, p. 55.

‡ Electrostatics, p. 317, § 400.

connecting them by any metallic conductor. By the passage of the electricity a portion of the electrolyte is decomposed, and the potential energy thus developed is equal to that possessed by the electricity while separated in the metals"*.

Professor Jenkin advocates essentially the same views:—"When two dissimilar metals are plunged side by side into a liquid such as water or dilute sulphuric acid, they do not exhibit *any* sign of electrification; the three materials remain at one potential, or nearly so. If while the two dissimilar metals are in the liquid they are joined by metallic contact to terminal pieces of one and the same metal, these terminal pieces will be brought to the same difference of potentials as that which would be produced by direct contact between the dissimilar metals"†. This amounts simply to saying that, as long as no wires are attached to the plates of a single cell, there is no difference of potential; but that when wires are joined on, the observed difference of potential is due to the contact of the *wire* with that metal plate to which it is dissimilar.

Again:—"When a single metal is placed in contact with an electrolyte, a definite difference of potentials is produced between them; zinc in water becomes negative, copper in water becomes negative, but less so than zinc. If, however, the two metals are plunged *together* into water, the copper, zinc, and water forming a galvanic cell, all remain at one potential, and no charge of electricity is observed on any part of the system." "If a piece of copper be now joined to the zinc, it (the copper) will become negative, and the other copper plate positive, the difference of potentials being that due to the direct contact between the zinc and piece of copper *only*, the water having the effect of simply conducting the charge from the zinc to the copper plate and maintaining them at one potential"‡.

The foundation for these statements is found apparently in an experiment due to Sir W. Thomson. He finds that if half-disks of zinc and copper be arranged under a movable metallic needle maintained at a high positive potential, if they are connected by a wire or by contact, the needle moves in such a way as to show that the copper is negative and the zinc positive; while if they are separated by a slight interval and connected by a drop of water, *no* difference of potential is observed. Professor Jenkin also lays great stress on the fact that, whereas copper in contact with zinc becomes negative, in a single cell with wires attached it is the wire attached to the zinc that shows a negative potential. This he holds to be conclusive that the junction of the wire with the zinc plate is the real seat of the electrical separation; although he admits that there may be a slight difference due to the liquid, and that different liquids may augment or decrease this difference.

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* Thermodynamics, p. 66, § 116.

† Electricity and Magnetism, p. 22.

‡ Electricity and Magnetism, p. 44.

In another place he says:—"If the voltaic theory of the cell were absolutely correct, the electromotive force of the cell would depend wholly on the plates in the electrolyte, and not at all on the solution employed to connect them"*. But it has been found that the potential series of the metals is slightly changed by the solution employed to join the plates: in order to account for this fact it is necessary to treat the voltaic theory as incomplete. He adds, however, that the potential series of the metals for water, dilute acids, and ammonic chloride do not differ so much as to invalidate the theory, although the series for alkaline sulphides is quite different and anomalous†.

4. When the two plates in one electrolyte are joined by a wire, or when simple cells are joined up in series and the circuit closed by a wire, we find it stated that there is a constant separation of the electricities at the point of contact of different metals and a constant recombination, attended with decomposi-

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* Electricity and Magnetism, p. 215.

† It may be remarked in passing, that this identity of the potential series for different acids may perhaps arise from a different cause, and not be altogether such a proof of the contact theory as Professor Jenkin concludes it is. Andrews has shown that, when *one* metal combines chemically with *different* acids, the amount of heat liberated is the same, or nearly so. Hence, if the metals be arranged in the order of their heat-producing power when combined with the same acid, that order will remain the same for most other acids. But the order is quite different when the metals are combined with sulphur or oxygen. It is true that this order is not the electrochemical one; but various causes may interfere to disturb it. At any rate it is sufficient to show that this fact of the partial identity of the potential series for different acids cannot by any means be claimed as conclusive of the contact theory. Moreover, although the *order* may be the same for the different liquids, we do not know that the *coefficients* are the same for each metal in every dilute acid.

tion, through the electrolyte. "Perhaps it is strictly accurate to say that the difference of potential is produced by the contact, and that the current which is maintained by it is produced by chemical action"*. And, lastly, that in a series of cells the electromotive force is due to the sum of the differences of potential produced by all the contacts.

The above quotations may be taken as affording the plainest notion of the new contact theory; and it will be seen that its fundamental propositions are briefly these:—

I. That two plates of different metals in one liquid are at the same potential when insulated and separated; *i. e.* there is *no* difference of potential due to chemical affinity.

II. In a cell series the gradual rise in potential, or the electromotive force, is due only to the dissimilar metallic contacts.

III. The chemical action in the battery is the result rather than the cause of the difference of potential, and is looked upon as an accompanying action rather than as the actual creator of the current, it having little or no share in the production of the difference of potential between the terminals.

These are, I venture to think, points not to be admitted as proved without further inquiry, and against which, as I shall hope to show, some grave if not insuperable objections may be urged, founded on other experimental evidence.

The first question to be settled is, then, whether in a series of cells the *whole* of the difference of potential between the terminals is due to the contacts, as above stated, or whether *any* portion is due to the tendency towards chemical combination existing between the metals and the electrolytes; and, as a consequence, whether in a single cell the plates are at the same potential or at different potentials, owing to the difference of chemical action upon them. Now I think this point will be sufficiently proved if we can establish by experiment, (i) that a battery of cells can be constructed without any dissimilar metallic contacts and with terminal plates of the same metal, and which shall yet exhibit difference of potential and continuous current; for if this is possible, it must follow that chemical affinity *alone* is capable of *creating* electromotive force as well as of maintaining a current, and that, in an ordinary cell-series, *some* part at least of the electromotive force is due to this cause, whilst the remainder is the result of the metallic contacts that may exist. Or (ii) if we can establish directly that the two plates in one cell are not at the same potential, as stated by more than one authority.

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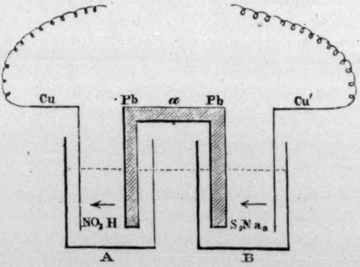
* Electricity and Magnetism, pp. 53-55.

With regard to the first point, it will be remembered that an old experiment of Faraday's proved that a current can be maintained and decomposition effected by a single cell where there is no dissimilar contact. It is not easy to see how this experiment can be explained by any form of contact theory; indeed it appears unanswerable. But in order to leave no point unsettled by experiment, it seemed desirable to try and arrange a series of cells in which all dissimilar contact was absent, so that the difference of potential due to chemical action might be separated from that due to the contacts and rendered visible by the electroscope.

It is obvious that we can make no attempt to do this unless we can in some way or other obtain a battery with terminals of the same metals; for otherwise the very junctions with the electroscope introduce what we want to eliminate, viz. dissimilar metallic contact. But the following is a method by which this can be accomplished. If plates of lead and copper be placed in nitric acid the lead is positive to the copper, since it is most acted upon; but if lead and copper be placed in solutions of alkaline persulphides, then the copper is most readily acted upon and is positive to the lead; that is, the positions are reversed.

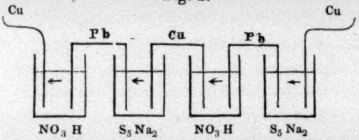
Now, if we place in a cell A dilute nitric acid and a copper and a lead plate, we cannot join up another cell of the same sort

Fig. 1.



in series without introducing contact. But if, instead of using a cell containing acid, we place next A a cell, B, containing sodic pentasulphide, and bend over the lead plate of A to dip into the liquid in B, and place in B also a copper plate, we shall then have two cells joined up in series without dissimilar contact and with similar metals for terminals; and yet the action of the liquids on the metals is such that in A the lead is positive to the copper Cu, and in B the copper Cu$'$ is positive to the lead. Hence there is a regular rise in potential in passing through the two cells; and on joining Cu Cu$'$ by a copper wire a current flows through both cells in the same direction, and the general effect is to urge round a current in the direction shown by the arrows. It is obvious that we need not limit ourselves to two cells. By forming a pile of alternate cells filled with acid and alkaline persulphide, connected by bent copper and lead plates alternately (fig. 2), we shall be able to accumulate difference of potential to any extent; and if the number of acid and alkaline cells be equal, we shall always end with a plate similar to that with which we began. Such a battery will exhibit a difference of potential between its two terminals when the circuit is opened, and will give a current when it is closed. In it we have nothing but chemical action to rely upon both for creating electromotive force and for maintaining the current. We have no dissimilar contacts; and as the terminal plates are similar, we can effect the junctions with the electroscope without introducing an unbalanced dissimilar contact. I have constructed such a battery of 60 cells; and by the kindness of Professor Guthrie, to whom my thanks are due, I have been permitted to compare its potential with that of a Daniell's cell, by means of a quadrant electrometer belonging to his laboratory. By this means it is at once seen that the difference of potential increases proportionally to the number of cells, the electromotive force of four cells being about equal to that of one Daniell. Joined up with a galvano-

Fig. 2.



meter it indicates a current, which, however, rapidly falls off in strength, owing to the formation of an insoluble cupric sulphide upon the copper plates. Joined up in opposition to a single Daniell's cell, with a galvanometer included in the circuit, I find that it requires from four to five cells to balance the force of the Daniell at first immersion; but after leaving it to work on short circuit for 2½ hours its electromotive force had fallen off 50 per cent.; it then required about 8 cells to bring the needle to zero. This gives for the electromotive force of two cells at first about ·5 of a volt; or the whole sixty cells are equal nearly to 15 Daniell's cells. It readily effects the decomposition of many electrolytes, and exhibits therefore every property of an ordinary cell-series. Above all, it will be noticed that since there is a regular rise in potential in passing from cell to cell, and as all parts of each plate must be at the same potential, that rise can only take place at the surfaces where the active metals are in contact with the electrolyte (that is, at the seat of the chemical action), and that therefore two metals in one electrolyte *cannot* be at exactly the same potential. But I find that more direct evidence still of this fact is to be found in an experiment of Faraday's, which seems to have escaped the notice of the contact theorists.

In his 'Experimental Researches' he gives the following fact. "I took a voltaic apparatus, consisting of a single pair of large plates, namely a cylinder of amalgamated zinc and a double cylinder of copper. These were put into a jar containing dilute sulphuric acid, and could at pleasure be placed in metallic communication by a copper wire connecting the two plates. Being thus arranged, there was no chemical action whilst the plates were not connected; on *making* the contact a spark was obtained. In this case it is evident that the first spark must have occurred before metallic contact was made, for it passed through an interval of air; and also that it must have tended to pass before the electrolytic action began, for the latter could not take place until the current passed, and the current could not pass before the spark appeared." "Hence," he says, "I think there is sufficient proof that the zinc and water were in a state of powerful *tension* previous to the actual contact"*. It is difficult to reconcile this with the experiment of the half disks and drop of water made by Sir W. Thomson. But, at any rate, a consideration of the whole of the facts would seem to point out that the only safe conclusion is, that in any series of cells of any sort the electromotive force is a complex effect, being due to the algebraical sum of all the differences of potential due to dissimilar contacts *plus* the algebraical sum of the differences of potential due to the chemical affinities of the metals and electrolytes *minus* any opposing force due to polarization &c.; and that so far from being the exclusive cause, the contacts can only be said strictly to have a share in producing the difference of potentials between the extremities of a battery*. And, lastly, we may with advantage compare the statements of the contact theory with certain other well-ascertained facts. Such statements, for instance, as these:—"If we close the circuit by connecting the metals by a wire, we then have constant separation of electricities at the point of contact of different metals, and constant recombination attended with decomposition through the electrolyte"†. "The electricities separated at the metallic junctions recombine through the water," "whilst the current flows the water is decomposed"‡,—which seem based on the assumption that the principal seat of the electrical actions is *not* to be looked for at the seat of the chemical actions. But, now, how does this fit in with those cases of electrochemical inversions noticed by De la Rive, where the direction of the current in a cell is *reversed* by simply diluting the electrolyte. Thus zinc is negative to tin in strong nitric acid, and mercury negative to lead; but in weak nitric acid the positions are reversed. Hence, if couples be formed of these metals in strong nitric acid, and the acid be gradually diluted, the current first ceases and then is reversed in direction.

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* Experimental Researches in Electricity, Series viii. ¶ 956.

Here, without altering the metallic junctions, we can at pleasure alter the direction of the current, and therefore also the direction of the fall in potential, since the current must flow from high to low potential. This seems conclusive that the chemical electromotive force must be even greater than the contact electromotive force. This reversal of the current, by changing the seat of the chemical activity, may be shown in another way, depending on the application of a very old principle. If plates of copper and clean iron be connected by copper wires with a galvanometer, and the iron rendered passive by immersion for a moment in strong nitric acid, then if these plates are plunged into dilute nitric acid the galvanometer indicates a strong current going through the cell from the copper to the iron. If they be removed for an instant and the iron plate touched, on again immersing the current is found to be reversed. Or we may again change the conditions, and notice that it is not sufficient to have merely two different metals and an electrolyte to form a cell. If plates of pure gold and platinum be placed in nitric acid, the most delicate galvanometer detects no current, and the same for many other pairs of metals and electrolytes.

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* Amounting in a Daniell's cell perhaps to 60 or 70 per cent. of the whole electromotive force.

† Tait, 'Thermodynamics,' § 116.

‡ Jenkin, 'Electricity and Magnetism,' p. 54.

Here we have contact of different metals producing its difference of potential; yet no current flows round "decomposing the electrolyte," as, according to the contact theory, it should do; but the instant we give play to chemical combination the ordinary results ensue. If the extremities of the copper wires from a galvanometer be attached to iron plates, and these plunged into separate cups of dilute nitric acid, on making connexion between the two cups by a bent iron plate dipping into each no current is detected. On making one limb of the connecting plate passive and re-immersing, a strong current is visible; and we find that we have the direction of the current completely under command by making any of the four plates more or less acted on than the other three.

If these experiments are to have any importance attached to them, it can scarcely be doubted that they land us in conclusions similar to the others, namely:—that we must look for the principal source of the electrical disturbance at that place where the greatest chemical activity is being brought into play; and that whereas contact of metals is in itself productive of definite electrical separation, there is in the battery another cause assisting in the production of difference of electrical potential between the terminals, viz. the potential chemical combination between the metals and electrolytes existing when the circuit is open—the energy of the current produced when the circuit is closed being, of course, the equivalent of this potential energy which disappears.

April 18, 1874.

II. *On Wind-pressure in the Human Lungs during Performance on Wind Instruments. By* Dr. W. H. Stone.

The object of these experiments was originally physiological. It had been stated by many writers that the forced expiration employed in playing tended to produce emphysema of the lungs; but the real amount of such pressure had never been measured.

The facts elicited had also an interest of a purely physical character, which was the principal cause of their being brought before this Society, although, the writer of the paper remarked, it was on the border-ground between two great subjects of study that new phenomena were often to be looked for.

The experiments were two in number. The first aimed simply at measuring, by means of a water-gauge, the extreme pressure which could be supported by the muscles of the lips, both in trained musicians and in persons unaccustomed to the continuous exercise of these organs. The difference between different individuals was very great, some untrained persons having naturally considerable muscular power. About 6 feet of water was the ordinary maximum when a small tube was inserted between the lips. When the lips were supported by a cupped mouthpiece, such as is used for brass instruments, a greater height of the column could be obtained. The great majority of untrained persons could not support more than three or four feet of water. It was to be noticed that the lip-muscles invariably gave way long before the expiratory power of the thoracic muscles was exhausted. By pinching the lips round the orifice of the tube with the hand, and thus preventing their yielding, a far higher column of water could be supported.

The second experiment consisted in introducing a small bent glass tube into the angle of the mouth, connected with a flexible tube passing over the shoulder. It was found that most instruments could be played as well with this addition as without it. It obviously established a communication between the cavity of the performer's mouth, and therefore of his thorax, and the pressure-gauge. The following Table was compiled from many observations on some of our principal English musicians. The person experimented on was placed with his back to the gauge, the small tube was inserted in his mouth, and he was directed to sound in succession the chief notes of his instrument. As soon as the tone became full and steady, the position of the water-gauge was noted. A fair "mezzo-forte" note was employed. Of course, by forcing the wind and overblowing the instrument, much greater pressure could be obtained; but those given here were sufficient to produce an average orchestral tone.

Oboe . . .	lower notes	9	inches;	highest	17 inches.
Clarinet . .	„	15	„	„	8 „
Bassoon . .	„	12	„	„	24 „
Horn . . .	„	5	„	„	27 „
Cornet . . .	„	10	„	„	34 „
Trumpet . .	„	12	„	„	33 „
Euphonium .	„	3	„	„	40 „
Bombardon .	„	3	„	„	36 „

It is to be noticed that the clarinet, in this as in some other respects, differs from its kindred instruments—and also that most of the pressures are small, not exceeding or, indeed, attaining the pressure of a fit of sneezing or of coughing. They are therefore very unlikely to injure the lungs, or to produce the emphysema erroneously attributed to them.

May 9, 1874.

III. *On some Physical Properties of Ice; on the Transposition of Boulders from below to above the Ice; and on Mammoth-remains. By* JOHN RAE, *M.D., LL.D., &c.*

Is the ice formed on salt water fresh? or, in other words, if ice formed on the sea is thawed, will the water obtained thereby be fresh?

For a number of years past I have spoken with many persons on the above subject; and seldom, if ever, have I found a single individual who did not say that the ice of the sea was fresh.

Some of these gentlemen are known in the scientific world; and many of them supported their opinions by quoting the highest written authorities on the subject, chiefly Tyndall's 'Forms of Water,' p. 132, par. 339, which tells us that "even when water is saturated with salt, the crystallizing force studiously rejects the salt, and devotes itself to the congelation of the water alone. *Hence the ice of sea-water, when melted, produces fresh water.*"

It is the sentence in italics to which I wish to draw particular attention.

It would be the extreme of folly and presumption on my part to question the correctness of results obtained by scientific men in their experiments in freezing small quantities of sea-water by artificial means, more especially those of the distinguished gentleman whose name I have mentioned, who, in addition to holding the high position of being one of our greatest authorities in all that relates to physical science, possesses the rare gift of being able to communicate his knowledge in such plain, clear, and forcible language, illustrated by admirable experiments, as to make his meaning fully understood, even by those who had previously been perfectly ignorant of the subject.

It is only where I have had opportunities of witnessing the action of cold carried on in a manner which may have been denied to the scientific man, that I venture to differ from him; and it is in this way that the conviction has been forced upon me, that the ice of sea-water if melted *does not* produce fresh water.

Before entering upon this subject, however, let me say a word or two on the first part of the quotation I have given.

If a saturated solution of salt is frozen, and the ice so formed is fresh, it is evident that the salt that has been "rejected" must be deposited or precipitated in a crystalline or some other solid form, because the water, if any, that remains unfrozen, being already saturated, can hold in solution no more salt than it already contains.

Could not salt be obtained readily and cheaply by this means from sea-water in cold climates?

During several long journeys on the Arctic coast, in the early spring before any thaw had taken place, the only water to be obtained was by melting snow or ice. By experience I found that a kettleful of water could be obtained by thawing ice with a much less expenditure of fuel, and in a shorter time, than was required to obtain a similar quantity of water by thawing snow. Now, as we had to carry our fuel with us, this saving of fuel and of time was an important consideration, and we always endeavoured to get ice for this purpose. We had another inducement to test the sea-ice frequently as to its freshness or the reverse.

I presume that almost every one knows that to eat snow when it is very cold, tends to increase thirst, whereas a piece of ice in the mouth is refreshing and beneficial, however cold it may be; we were consequently always glad to get a bit of fresh ice whilst at the laborious work of hauling our heavy sledges; yet with these strong inducements we were never able to find sea-ice, *in situ*[*], either eatable when solid or drinkable when thawed, it being invariably much too salt. The only exception (if it may be called one) to this rule, was when we found rough ice, which, from its wasted appearance and irregular form, had evidently been the formation of a previous winter. This old ice, if projecting a foot or two above the water-level, was almost invariably fresh, and, when thawed, gave excellent drinking-water. It may be said that these pieces of fresh ice were fragments of glaciers or icebergs; but this could not be so, as they were found where neither glaciers nor icebergs are ever seen.

How is this to be accounted for? Unfortunately I have only a theory to offer in explanation.

When the sea freezes by the abstraction of heat from its surface, I do not think that the saline matter, although retained in and incorporated with the ice, assumes the solid state, unless the cold is very intense, but that it remains fluid in the form of a very strong brine enclosed in very minute cells. So long as the ice continues to float at the same level, or nearly the same level, as the sea, this brine remains; but when the ice is raised a little above the water-level, the brine, by its greater specific gravity, and probably by some solvent quality acting on the ice, gradually drains off from the ice so raised; and the small cells, by connecting one with another downwards, become channels of drainage.

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[*] What I mean by ice *in situ* is ice lying flat and unbroken on the sea, as formed during the winter it is formed in.

There may be several other requisites for this change of salt ice into fresh, such as temperature raised to the freezing-point, so as to enable the brine to *work out* the cell-walls into channels or tubes—that is, if my theory has any foundation in fact, which may be easily tested by any expedition passing one or more winters on the Arctic, or by any one living where ice of considerable thickness is formed on the sea, such as some parts of Norway.

All that is required, as soon as the winter has advanced far enough for the purpose, is to cut out a block of sea-ice (taking care not to be near the outflow of any fresh-water stream) about 3 feet square, remove it from the sea to some convenient position, test its saltness at the time, and at intervals repeat the testing both on its upper and lower surfaces, and observe the drainage if any.

The result of the above experiment, even if continued for a long while, *may* not be satisfactory, because the fresh ice that I have described must have been formed at least twelve months, perhaps eighteen months, before.

The Transposition of Boulders from below to above the Ice.

When boulders, small stones, sand, gravel, &c. are found lying on sea-ice, it is very generally supposed that they must have rolled down a steep place or fallen from a cliff, or been deposited by a flow of water from a river or other source. There is, however, another way in which boulders &c. get upon floe-ice, which I have not seen mentioned in any book on this subject.

During the spring of 1847, at Repulse Bay on the Arctic shores of America, I was surprised to observe, after the thaw commenced, that large boulders (some of them 3 or 4 feet in diameter) began to appear on the surface of the ice; and after a while, about the month of July, they were wholly exposed, whilst the ice below them was strong, firm, and something like 4 feet thick.

There were no cliffs or steep banks near from which these boulders could have come; and the only way in which I could account for their appearance, was that which by subsequent observation I found to be correct.

On the shores of Repulse Bay the rise and fall of the tide are 6 or 8 feet, sometimes more. When the ice is forming in early winter, it rests, when the tide is out, on any boulders &c. that may be at or near low-water mark. At first, whilst the ice is weak, the boulders break through it; but when the ice becomes (say 2 or 3 feet) thick, it freezes firmly to the boulder, and when the tide rises, is strong enough to lift the boulder with it. Thus, once fastened to the ice, the stone continues to rise and fall with the rise and fall of each tide, until, as the winter advances, it becomes completely enclosed in the ice, which by measurement I found to attain a thickness of more than 8 feet.

Small stones, gravel, sand, and shells may be fixed in the ice in the same way.

In the spring, by the double effect of thaw and evaporation, the upper surface of the ice, to the extent of 3 feet or more, is removed, and thus the boulders, which in autumn were lying at the bottom of the sea, are now on the ice, while it is still strong and thick enough to travel with its load, before favourable winds and currents to a great distance.

The finding small stones and gravel on ice out to sea does not always prove that such ice has been near the shore at some time or other.

I have noticed that wherever the Walrus in any numbers have been for some time lying either on ice or rocks, a not inconsiderable quantity of gravel has been deposited, apparently a portion of the excreta of that animal, having probably been taken up from the bottom of the sea and swallowed along with their food.

Mammoth-remains. The position in which their Skeletons are found, &c.

In Lyell's 'Principles of Geology,' vol. i. p. 185, we read:—"In the flat country near the mouth of the Yenesei river, Siberia, between latitudes 70° and 75° north, many skeletons of mammoths, retaining the hair and skin, have been found. The heads of most of these are said to have been turned to the south."

As far as I can find, the distinguished geologist gives no reason why the heads of the mammoths were turned to the south; nor does he say all that I think might be said of the reasons why, and the means by which the skins have been preserved for such a long period of time.

Having lived some years on the banks of two of the great rivers of America, near to where they enter Hudson's Bay, and also on the M'Kenzie, which flows into the Arctic Sea, I have had opportunities of observing what takes place on these streams, all of which have large alluvial deposits, forming flats and shallows at their mouths.

What I know to be of common occurrence in these rivers may, if we reason by analogy, have taken place in ancient times on the great rivers of Siberia, making due allowance for the much higher northern latitude to which these streams run before reaching the sea, and for the difference in size of the fauna that used to frequent their banks.

When animals, more especially those having horns, tusks, or otherwise heavily weighted heads, are drifting down a river, the position of the bodies may lie in any direction as regards the course of the stream, as long as they are in water deep enough to float them; but the moment they get into a shallow place, the head, which sinks deepest (or, as sailors say, "draws most water"), takes the ground, whilst the body, still remaining, afloat, swings to the current, just as a boat or ship does when brought to anchor in a tideway.

It is probable that the mammoths, having been drowned by breaking through the ice or in swimming across the river in spring when the banks were lined with high precipitous drifts of snow, which prevented them from getting out of the water or killed in some other way, floated down stream, perhaps for hundreds of miles, until they reached the shallows at the mouth, where the heads, loaded with a great weight of bone and tusks, would get aground in 3 or 4 feet of water, whilst the bodies still afloat would swing round with the current as already described.

The Yenesei flows from south to north, so the heads, being pointed up stream, would be to the south*.

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* Not many years ago, when buffalo were very abundant on the Saskatchewan, hundreds of them were sometimes drowned in one season whilst

Supposing, then, these bodies anchored as above in 3 or 4 feet water; as soon as the winter set in, they would be frozen up in this position. The ice in so high a latitude as 70° or 75° north would acquire a thickness of 5 or 6 feet at least, so that it would freeze to the bottom on the shallows where the mammoths were anchored. In the spring, on the breaking up of the ice, this ice being solidly frozen to the muddy bottom, would not rise to the surface, but remain fixed, with its contained animal remains, and the flooded stream would rush over both, leaving a covering of mud as the water subsided.

Part of this fixed ice, but not the whole, might be thawed away during summer; and (possibly, but not necessarily) next winter a fresh layer of ice with a fresh supply of animal remains might be formed over the former stratum; and so the peculiar position and perfect state of preservation of this immense collection of extinct animals may be accounted for without having recourse to the somewhat improbable theory that a very great and sudden change had taken place in the climate of that region.

I have seen at the mouth of Hayes River in America animals frozen up as above described; but as the latitude of this place is only 57° north, the fixed ice usually wholly disappears before the next winter sets in, and liberates the animals shut up in it; but when the rivers reach the sea, as some of those of Siberia do, 1000 or 1200 miles further to the north, it may be fairly assumed that a large part of this fixed ice, protected as it would be by a layer of mud, might continue unthawed.

IV. *On the Fall in Pitch of Strained Wires through which a Galvanic Current is passing.* *By* Dr. W. H. Stone.

The object of this paper was to apply the vibrations of sound to the measurement of electrical currents, and to distinguish what was due to heating-effects from those caused by alteration of elasticity.

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swimming across the river; and many reindeer, moose, and other animals are annually destroyed in this way in other large American rivers.

Sir Charles Lyell mentions a number of yaks being seen frozen up in one of the Siberian rivers, which, on the breaking up of the ice in spring, would be liberated and float down the stream.

Strings of brass and steel, such as are used for pianofortes (No. 16 gauge), were stretched, by means of wrest-pins, across a resonant box, over bridges surmounted by brass bearings, and tuned to unison. On passing a current from two or more Grove's batteries through them, a very marked fall in pitch was obtained. The vibrating string being 24 inches long, and tuned to two-foot C, the tone sank above a fourth in steel and a major third in brass.

This result being a compound of actual lengthening by heat and of other causes, it was, in a second experiment, endeavoured to eliminate the former element by straining similar strings between the same bridges by means of a weight. This was attached to the arm of a bent lever, to the short end of which the string was made fast. By shifting the position of a four-pound weight along the arm, very accurate unison, or definite periodicity of beats could be obtained. When the current from the battery was passed through this string, free to expand by the falling of the weight, and therefore at a constant tension, a fall of pitch was still noticed. There was also a very marked loss of tone, which, on approaching a red heat, amounted to total extinction of sound.

A third experiment exhibited the changes of electrical resistance in a wire subjected to variations of strain. The wire was accurately balanced against another resistance in a Wheatstone's bridge, and the spot of light from a mirror-galvanometer joining the two circuits thrown on the screen. On suddenly increasing the tension and raising the musical pitch of the string, the galvanometer was visibly deflected. This was not an effect of heat (since the balance had been brought about during the passage of the current), and must be due to altered molecular state caused by the strain.

It was incidentally noticed that, when beats were produced by two strings on the same sonometer, they continued to be sensible to the touch by laying the hand on the instrument long after, from diminution of amplitude in the vibration, or from slowness in the beats themselves, they had ceased to be audible. This afforded a good demonstration of the continuity of sensation in touch and hearing.

V. *On a simple Method of Illustrating the chief Phenomena of Wave-Motion by means of Flexible Cords. By the late* W. S. Davis, *F.R.A.S., Derby.*

[With a Plate.]

The simple methods about to be described, of exhibiting the chief phenomena of wave-motion, were suggested during some experiments lately made by the author on the refraction of liquid waves*. These experiments consisted in the production of waves on the surfaces of two liquids of different densities, lying side by side: on agitating the surface of either liquid, waves were produced which passed from one liquid to the other, at the same time undergoing changes in amplitude, length, and form of front. In preparing diagrams to represent these phenomena it became necessary to make drawings of vertical sections through the two liquids, perpendicular to their line of separation.

The appearance presented by the sinuous lines on these diagrams immediately suggested that a similar appearance could be exhibited by means of waves on flexible cords. India-rubber tubes, variously suspended, and both empty and loaded, were tried without satisfactory success; the waves moved too quickly to be well observed, and the reflected waves interfered with the direct ones. Further experiments led the author to devise the simple apparatus now exhibited, which, however, has been made to serve for many other illustrations of wave-motion in addition to those it was at first intended to show.

The apparatus consists essentially of:—(1) a piece of stout board about 20 feet long and 9 inches wide, which should be painted black; and (2) three or four ropes, which must be both heavy and flexible: the ropes used by builders for securing their scaffolding have been found to answer very well, especially if they have been in use some time. To enable the eye to readily distinguish any particular rope when two or more are used together, it is well to cover the ropes with differently coloured fabrics, say red, blue, and green. A few other accessories are necessary, which will be described as they are required.

By means of this apparatus waves may be produced which

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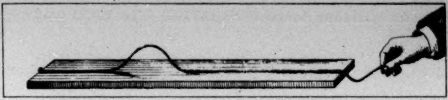
* See Brit. Assoc. Report, 1873.



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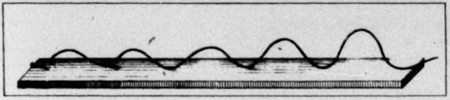

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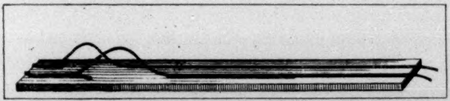

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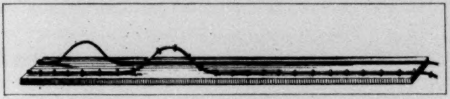

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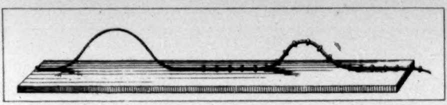

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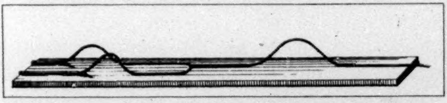

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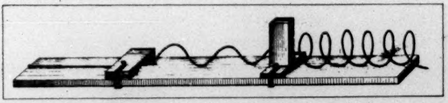

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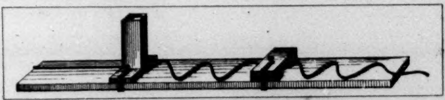

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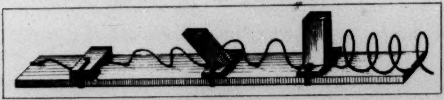

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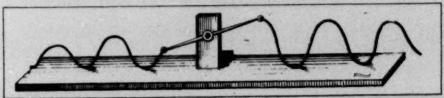

10.

move slowly enough to be readily examined by the eye. The chief phenomena of wave-motion which can be shown are as follows:—

1. *Transmission of a Wave.*—One end of a rope, a few feet longer than the board, is fixed to a hook at the end of the board. The free end of the rope is then taken in the hand, and, the the rope being quite slack, a sudden up-and-down movement of the hand is made. A protuberance is thus formed which moves very slowly along the rope, presenting the appearance shown in Plate I. fig. 1*.

A single up-and-down movement produces a wave consisting of a crest only, the trough being suppressed by the board; if, however, with the rope very slack, the hand be moved up and down very quickly and energetically, a series of waves, consisting of both crest and trough, are produced (fig. 2).

2. *Amplitude and Wave-length.*—Waves having any length, from 1 to 6 or 7 feet, and amplitudes of similar dimensions, are easily produced by properly controlling the rapidity and energy of the motion of the hand.

3. *Decrease of Intensity with Distance.*—This is illustrated by a succession of waves produced by the well-timed motion of the hand (fig. 2). The actual decrease of amplitude in this case is, of course, due to the loss of energy by friction, and not to lateral spreading.

4. *Relation of Velocity to Elasticity.*—Two similar ropes, one covered with red and the other with blue, are laid side by side along the board and fastened to hooks at one end. The free ends of the ropes are held in the hand, with the finger between them, and, care being taken that they are equally loose, the hand is moved up and down as usual. The result is that a wave of the same height and length is produced on each rope, and the two waves travel side by side to the ends of the ropes. The experiment is repeated with one rope somewhat tighter than the other, when the wave on the tighter rope is observed to travel faster than that on the looser one (fig. 3). On continuing to tighten the rope the velocity of the wave is more and more increased, and may be caused to reach the end of the rope a whole length or more before its fellow.

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* The *length* of the board in the figures is drawn to a much smaller scale than the other parts.

5. *Relation of Velocity to Density.*—To exhibit this relation a loaded rope is required. That now used has strung upon it a number of rings of lead cut from a leaden water-pipe; these are placed about 6 inches apart, and are covered with india-rubber bands to prevent their making unpleasant noise. The loaded and an unloaded rope are laid on the board side by side, and fixed at one end. Then, taking care the tension is equal in the two ropes, waves are simultaneously generated on them, as before described. It is then observed that the wave on the loaded rope lags considerably behind the other (fig. 4). By sufficiently tightening the loaded rope the velocity of its waves may be made equal to, or even greater than that of the waves of the unloaded rope. This may be used to explain why the velocity of sound in water is greater than in the much less dense medium, air.

6. *Transmission of Waves from one Medium to another of different Density.*—The loaded cord is attached end to end to one much lighter than itself; the united cords are laid on the board with the splice at about the middle of its length. Then, fastening the end of the lighter cord, waves are generated on the heavier one. These waves pass onwards to the lighter cord, on reaching which they acquire greater amplitude, velocity, and length (fig. 5). If the heavier cord be fixed and waves be generated on the lighter one, the reverse changes to those just stated occur on the waves reaching the heavier cord. It is an interesting experiment to transmit waves along a succession of three or more cords alternately heavy and light. With three cords joined end to end, the middle one being heavier than the others, a good illustration is produced of the changes of velocity, length, and amplitude which ætherial waves undergo in passing perpendicularly through a medium with parallel faces.

7. *Separation of a Wave into two or more smaller Waves.*—A single cord extending half the length of the board is joined to a double one extending the other half. Waves are transmitted from the single cord to the double one; on reaching the latter each wave divides in two, one wave traversing one part of the double cord, and the other wave the other part. By giving each part of the double cord a different tension, the velocity of the waves will be different in each (fig. 6). The waves on the double cord may be made to move in planes at right angles to each

other by the use of proper guides, thus furnishing an illustration of some of the phenomena of double refraction.

8. *Superposition and Interference.*—The same arrangement is used as in 7, but the waves are transmitted from the double cord to the single one. With equal tension in each part of the double cord, the waves simultaneously produced on each part run side by side until they enter the single cord, when they are superposed and produce a wave of double amplitude. One half of the double cord may be tightened until its wave reaches the single cord half a wave's length before the wave on the other half, when interference occurs, there being little or no lateral motion to be observed in the single cord.

9. *Plane of Waves.*—In the experiments previously described the waves were transmitted in a vertical plane; but by properly directing the motion of the hand, the waves may be transmitted in planes variously inclined to the board, or in a plane parallel with it. Waves in space of three dimensions, corresponding to circularly polarized light, are produced by rapidly and regularly moving the hand in a circle, the cord then taking the form shown at the right of figs. 7 and 9.

10. *Polarization.*—A series of flat boards are used as guides, which are clamped on the long board. These are shown in figs. 7, 8, 9. The vertical and oblique guides are each in two pieces, which are so approximated to each other as to just allow the cord to move freely between them. The horizontal guide is in one piece only. The vertical and horizontal guides being fixed as shown in figs. 7 and 8, waves in a vertical plane are transmitted from that end of the rope nearest the vertical guides; the waves then pass freely through the vertical guides, but are completely stopped by the horizontal one. Waves in a horizontal plane transmitted from the other end of the apparatus pass the horizontal guide, but are stopped by the vertical ones (fig. 8). Waves in an oblique plane transmitted from either end are resolved by the nearest guide into a component in its own plane and a component at right angles which is suppressed; the former passes on and is stopped by the next guide. Circularly polarized waves on reaching the guides are similarly resolved (fig. 7).

11. *Depolarization.*—A pair of oblique guides are required in addition to those described in 10. The arrangement of these is

shown in fig. 9, which needs no further explanation. The waves are supposed to proceed from right to left. With a single cord as in fig. 9, or with a partly double one as in fig. 6, an endless variation of experiments relating to polarization may be produced.

12. *Radiation and Absorption.*—A rod of iron about 2 feet in length, having an eye at the centre and at each end, is fixed by means of a screw or pin through the central eye to an upright support of wood clamped at about the middle of the board (fig. 10). The iron rod must be able to rotate freely about the pin in a vertical plane parallel to the board, but in no other plane. Attaching a cord to one end of the iron rod and continuing it to the end of the board, a series of properly timed waves are sent along it, when the rod vibrates in synchronism with the waves. If a second cord be attached to the other end of the rod and waves be transmitted as before, the vibrations of the rod set up waves in this cord which correspond in period and length to those on the first cord, thus furnishing an illustration of the reciprocity of radiation and absorption.

The author has reason to think that, as nearly all the above-described illustrations have been devised during the last twelve months, the method is capable of much further development and greater perfection.

May 23, 1874.

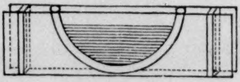
VI. *Glass Cell with Parallel Sides.*

By F. Clowes, *Esq., B.Sc., F.C.S.*

The following method has proved very convenient for making a glass cell, which may be readily fitted up from ordinary laboratory apparatus, and may also be rapidly taken to pieces for the purpose of being cleansed.

A piece of india-rubber tubing with stout walls, or, better, a length of solid rubber, is placed in the form of a letter U between two plates of glass, the ends of these plates being then firmly held together by slipping over them stout india-rubber

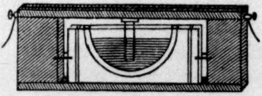
Fig. 1.



rings. A glass cell is thus obtained, the parallel faces of which are formed by the glass plates, whilst its thickness, depth, and length can be suitably varied by the stoutness and length of the rubber tube and the shape which this tube is made to assume.

With a glass cell of the size of an ordinary magic-lantern slide (fig. 1), the difference in specific gravity between hot and cold water* may be well shown upon the screen by a magic lantern, the liquid admitted by a pipette being preferably tinged by dissolving in it a crystal of potassium permanganate; and the convective currents occurring in the mass of a liquid may be thrown upon the screen by passing a galvanic current through a fine platinum wire stretched between two thick copper wires beneath the surface of the liquid in the cell: these currents are rendered much more evident by allowing the platinum wire to be immersed in a stratum of potassium-permanganate solution which has been cautiously introduced beneath the water by means of a pipette dipping to the bottom of the cell.

Fig. 2.



A smaller cell made to fit into the wooden frame of a lantern-slide (fig. 2), which has attached to it platinum wires connected by copper wires and binding-screws with a galvanic battery, serves to project electrolytic decompositions upon the screen. Perhaps the most beautiful appearance is that presented by the crystallization of the metal from a solution of lead-acetate which is undergoing electrolysis†.

In order that the cell may be water-tight, it is necessary that the india-rubber rings should exert a somewhat powerful compression; but even under favourable circumstances slight leakage is liable to occur in about half an hour after the cell has been filled; this, however, would allow ample time for the display of any of the phenomena above alluded to. Rings cut from large-sized india-rubber tubing have been found well adapted for the construction of small cells.

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* See Tyndall's 'Heat, a Mode of Motion,' pp. 173 and 174.

† Mr. W. Crookes, F.R.S., suggests the electrolysis of solution of thallium sulphate as furnishing a still more beautiful example of crystallization.

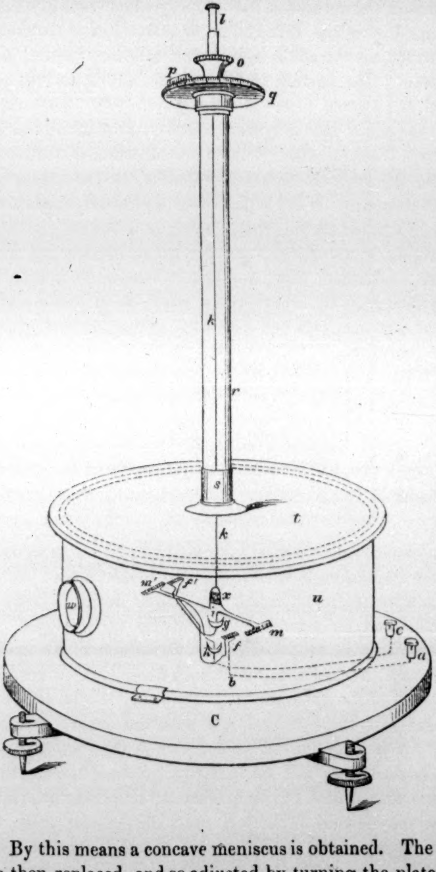
VII. *On an Absolute Galvanometer.*

By FREDERICK GUTHRIE.

MESSRS. ELLIOTT have constructed for me a galvanometer which will, I believe, be found to possess for some purposes certain advantages over those at present in use. Its principle depends upon the measurement of the current-strength by the measurement of the mechanical force necessary to bring to a given distance of one another two electromagnets, which are excited by the current in such a fashion that they repel one another.

The current enters at *a* by the screw-clamp; thence it passes beneath the circular wooden stand C along the copper wire *a b*. It rises vertically and coils round a soft iron mass *f*, which lies horizontal and tangential to the axis of the instrument. It passes down and across the centre of the board, then rises and coils round a soft iron mass f', exactly similar and similarly placed to *f*, but on the opposite side of the instrument. Having encircled f', the current-bearing wire again descends, and carries a mercury-cup *g*, through whose bottom it passes, and which is exactly in the axis of the instrument. The current then leaves the mercury by the wire *i*, which dips into it. It then traverses the wire around the iron, *m*. Thence it crosses the instrument and forms a spiral around m', after which it passes into the mercury-cup *h*, and so to the binding-screw *c*. The spirals are such that there is repulsion between *f* and *m*, and also between m' and f'. It is seen that the magnetic pair $f f'$ is fixed. The pair $m m'$ is movable about a vertical axis. The system $m m'$ is hung by a metal or glass thread *k* from the rod *l*, which works stiffly through the nut *o*. The latter carries an arm and vernier, *p*, which slides over the graduated head, *q*. The scale, nut, &c. are supported on the glass tube *r*, which is fastened by the cap *s* on to the plate-glass disk *t*, which rests upon the top of the glass cylinder *u* clamped upon the wooden base *c* resting on levelling-screws. In the side of *u* is a plate-glass window, *w*, through which a vertical line of light may be focused upon *x* (a mirror fastened to the $m m'$ system), and thence thrown upon a scale in the manner which is now so often employed.

A word or two about the way in which the instrument is used. The upper plate *t* and the system $m m'$ are removed by lifting *r*. The edge of *u* is rubbed with beeswax to prevent *t* from slipping upon it. The copper wires penetrating the cups are amalgamated and a little mercury poured in. Amalgamated thin platinum-foil is then pressed into the cups, and mercury is poured upon this. By this means a concave meniscus is obtained. The upper part is then replaced, and so adjusted by turning the plate *t* and the cylinder *u* that the mirror *x* is parallel to the window *w*, when the axis of *m m'* makes an angle of about 15° with that



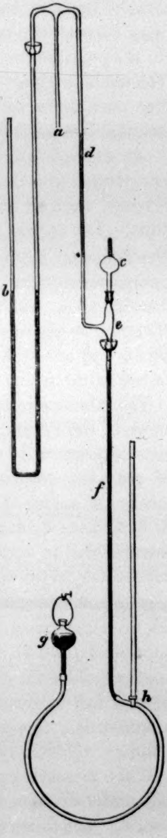
of ff'. The rod l is adjusted so that the wires of $m\,m'$ just touch the mercury; and by the levelling-screws k is so swung that m and f, and also m' and f', are exactly opposite to one another and the wires in the centres of the mercury-cups. A slit of light is then sent through w, reflected on to a screen, and the head o is then turned till the slit is split by an arbitrary vertical line on the screen. The reading of p is then noted. A current passing through the system forces $m\,m'$ away from ff'. Turn the head o until the slit of light is again brought to the mark on the screen. The angle through which it must be turned is directly proportional to the magneto-repulsion—that is, to the square of the current-strength. Many of the laws of electrodynamics may be readily illustrated by this instrument; and not only may different currents be compared with the greatest accuracy, but the absolute mechanical magneto-value of the current may be at once arrived at. By bringing the repellent magnets always to the same distance from one another, a whole class of sources of error is removed.

June 13, 1874.

VIII. *Apparatus for Measurement of Low Pressures of Gas.* By Professor M'Leod, *Indian Civil-Engineering College, Cooper's Hill.*

This apparatus was devised for estimating the pressure of a gas when its tension is so low that the indications of the barometer cannot safely be relied on, unless indeed a very wide barometer and an accurate cathetometer be employed. The method consists in condensing a known volume of the gas into a smaller space and measuring its tension under the new conditions.

The form of the apparatus is the following:—The tube a communicates with the Sprengel, and with the apparatus to be exhausted; b is a siphon-barometer with a tube about 5 millimetres in diameter; and the principal parts of the measuring-apparatus consist of c, a globe of about 48 cubic centims. capacity with the volume-tube at the top, and d the pressure-tube; these two are exactly of the same diameter, to avoid error from capillarity. The tube at the bottom of the globe is ground into a funnel-shaped portion at the top of the wide tube e; and to the side of the latter the pressure-tube d is joined. The volume-tube at the top of the globe is graduated in millimetres from above downwards, the lowest division in this particular apparatus being 45; the pressure-tube *d* is also graduated in millimetres, the 0 being placed at the level of the 45th division on the volume-tube. A ball-and-socket joint connects the bottom of *e* with a vertical tube *f* about 800 millims. long, which is connected at its lower extremity by means of a flexible tube with the mercury-reservoir *g*; a stopcock at *h* permits the regulation of the flow of mercury into the apparatus: this may be conveniently turned by a rod, so that the operator may watch the rise of the mercury through a telescope and have the stopcock at the same time at command.



The volume-tube was calibrated in the usual way, by introducing weighed quantities of mercury into it, and making the necessary corrections for the meniscus. The capacity of the volume-tube, the globe, and upper part of the tube *e* was determined by inverting the apparatus and introducing mercury through *e* until the mercury flowed down the pressure-tube; the weight of this quantity of mercury, divided by the weight of that contained in the volume-tube, gives the ratio between the volumes; in the present case it is 1 to 54·495. While the apparatus is being exhausted, the reservoir *g* is lowered so as to prevent the mercury rising out of the tube *f*; but when it is desired to make a measurement of the pressure, the reservoir is raised and the mercury allowed to pass through the stopcock *h*. On the mercury rising into the tube *e* it cuts off the communication between the gas in the globe and that in the rest of the apparatus. Ultimately the whole of the gas in the globe is condensed into the volume-tube; and its tension is then found by measuring the difference of level between the columns of mercury in the volume- and pressure-tubes. On dividing this difference by the ratio between the capacities of the globe and volume-tube, a number is obtained which is approximately the original pressure of the gas; this number must now be added to the difference between the columns, since it is obvious that the column in the pressure-tube is depressed by the tension of the gas in the remaining part of the apparatus; on dividing this new number once more by the ratio between the volumes the exact original tension is found.

An example will best illustrate this. A quantity of gas was compressed into the volume-tube, and the flow of mercury was arrested when its surface reached the lowermost division on the tube. The volume was then $\frac{1}{54{\cdot}495}$ of its original volume, and the difference between the levels of the mercury in the volume- and pressure-tubes was 66·9 millims.; this number, divided by 54·495, gives 1·228 as the approximate pressure. 1·2 must therefore be added to the observed column, which thus becomes 68·1; and on dividing by 54·495, the number 1·2497 is obtained as the actual pressure.

The relations existing between the contents of the other divisions of the volume-tube and the total contents of the globe were determined by measuring the tensions of the same quantity of gas when compressed into the different volumes. By this means the values of the divisions 40, 35, 30, 25, 20, 15, 10, 9, 8, 7, 6, 5, 4, 3, and 2 have been found; the experimenter is thus enabled to employ a division suitable to the quantity of gas with which he has to deal. The smallest division contains only $\frac{1}{1492{\cdot}35}$ of the globe; consequently when a quantity of gas has been condensed into this space, its original tension will be multiplied 1492·35 times. In one case an amount of gas, which originally filled the globe, exhibited a pressure of only ·5 millim. when it had been compressed into the smallest division of the volume-tube; this indicates an original pressure of only ·00033 millim.

When measuring a tension, it is advisable to make two readings under different condensations, and to take the mean of the results. The following will give some notion of the precision attainable:—

I. At division 5 ·0225 }
„ 2 ·0235 } Mean ·0230.

Remeasured.

At division 5 ·0228 }
„ 2 ·0236 } Mean ·0232.

II. Barometer 0 millim.:—

At division 10 ·1985 }
„ 5 ·1980 } Mean ·1982.

Remeasured.

At division 10 ·1953 }
„ 5 ·1967 } Mean ·1960.

III. Barometer 0·6 millim.:—

At division 15 ·5488 }
„ 10 ·5488 } Mean ·5492.
„ 6 ·5501 }

Remeasured.

At division 15 ·5464 }
„ 10 ·5464 } Mean ·5469.
„ 6 ·5480 }

IV. Barometer 1 millim.:—

At division 20 1·2042 }
„ 15 1·2069 } Mean 1·2055.

Remeasured.

At division 20 1·2082 }
„ 15 1·2099 } Mean 1·2090.

V. Barometer 1·5 millim.:—

At division 30 1·9139 }
„ 25 1·9080 } Mean 1·9109.

Remeasured.

At division 30 1·9041 }
„ 25 1·9039 } Mean 1·9040.

VI. Barometer 2·1 millims.:—

At division 35 2·6017 }
„ 30 2·6073 } Mean 2·6045.

Remeasured.

At division 35 2·6160 }
„ 30 2·6220 } Mean 2·6190.

1

It may be mentioned incidentally that connexions for apparatus may be conveniently made by means of ball-and-socket joints of glass. The ball is made by thickening a piece of tube in the blowpipe-flame, and the socket by cutting in half a thick bulb blown on a glass tube. The ball is then ground into the socket by means of emery and solution of soda, and afterwards polished with rouge and soda solution. When slightly greased and with a small quantity of mercury in the cup, a joint is obtained which is perfectly air-tight and flexible*.

IX. *On a simple Arrangement by which the Coloured Rings of Uniaxial and Biaxial Crystals may be shown in a common Microscope.* *By* Dr. W. H. STONE.

THE author was not aware that any arrangement had been hitherto supplied to the ordinary microscope other than an extra top to the eyepiece containing a supplementary stage and an analyzer. This could only be considered a clumsy expedient.

The objects to be obtained were clearly two:—first, to transmit the rays at considerable obliquity through the plate of crystal; secondly, to gather these up and form a real image within the tube of the microscope. Amici had accomplished this by a special combination of lenses which bears his name; it might, however, be done simply by placing a screwed diaphragm on the end of the upper draw-tube within the body of the microscope. The screw should be that ordinarily used for object-glasses. To this an object-glass of long focus was fitted, and another of higher magnifying-power in the usual place. The whole body was then drawn out and adjusted to a telescopic focus on a distant object. The lower objective formed the object-glass of the telescope, and the inner objective with the Huygenian eyepiece a compound ocular. On reinserting the

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* Since the above was written Dr. Sprengel has pointed out that Mr. Hartley (Proc. Roy. Soc. vol. xx. p. 141) has described as a "Sprengel joint" a connexion between two glass tubes made by grinding a conical tube into a conical cup and placing mercury or water in the cup. The difference between this and the one above mentioned is obvious: the former is quite rigid, the latter perfectly flexible.

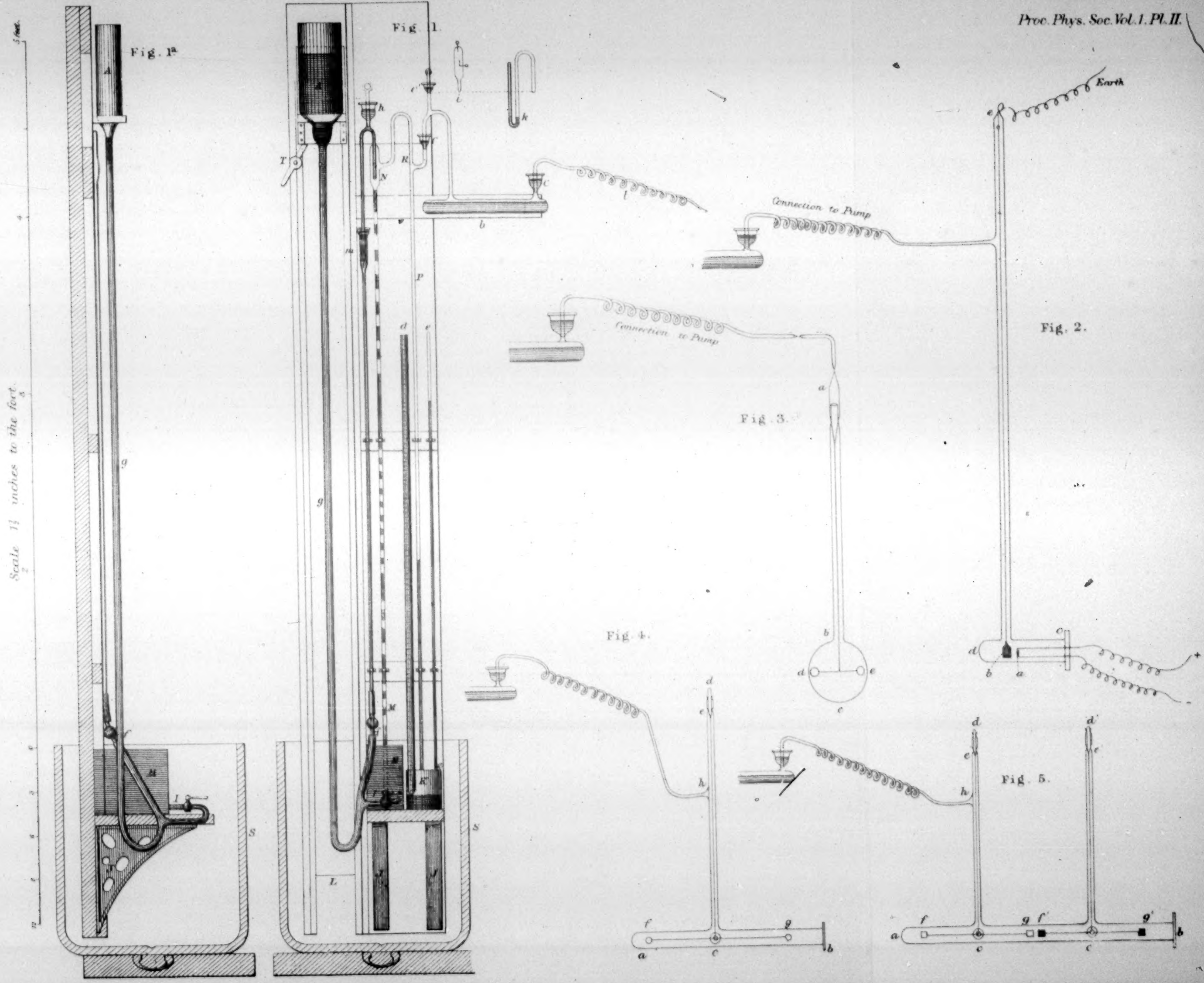
Fig. 1a.
Fig. 1.
Fig. 2.
Fig. 3.
Fig. 4.
Fig. 5.
Scale 1½ inches to the feet.
3 feet
4
3
2
A
g
H
I
S
T
h
c'
i
k
f
R
N
c
l
b
m
P
d
e
M
K'
J
L
Earth
Connection to Pump
Connection to Pump
a
+
−
d'
e'
f'
g'

body thus arranged, and illuminating the crystal on the stage with convergent light by means of a condenser, the rings and brushes could be perfectly seen. The whole double series of rings in a biaxial crystal of carbonate of lead was thus shown.

The condenser used was a "kettle-drum" of two plano-convex lenses. The objective on the nozzle of the microscope was a $\frac{2}{3}$ of Ross; that within the draw-tube a 3-inch objective of the same maker.

June 20, 1874.

X. *On Attraction and Repulsion accompanying Radiation.*

By William Crookes, *F.R.S. &c.*

[With a Plate.]

Before describing the apparatus and experiments which illustrate the attraction and repulsion accompanying radiation, it will perhaps be best to draw attention to the modification of the Sprengel pump which has so materially assisted me in this investigation.

Fig. 1 (Plate II.) shows the pump as now in use. Working so much with this instrument, I have endeavoured to avoid the inconveniences attending the usual mode of raising mercury from the lower to the upper reservoir. The mercury is contained in a closed glass reservoir A, perforated with a fine hole at the top. This reservoir is attached to a block capable of free movement in a vertical line and running in grooves, and connected with the lower reservoir by a flexible tube *g*. This tubing is specially made to stand a considerable pressure of mercury. It consists of a double thickness of india-rubber tubing enclosing a canvas tube in the centre, the whole being vulcanized together.

When the whole of the mercury has run through the pump, the reservoir and slide can be lowered by liberating a detent, T, and letting it descend to the block L. H is a glass reservoir which receives the mercury after flowing through the pump. When the reservoir A is emptied and has been lowered to the block L, the mercury from H is admitted into A by opening the tap I. At *f* is another tap, of platinum, to regulate the flow of mercury through the pump. *c*, *c*, *c'* are mercury joints, it being inconvenient to have the apparatus in one piece of tubing, and not always possible to seal the different portions together by fusion. *e e* is a barometer dipping into the same vessel as the gauge-barometer P, the two thus forming a differential system by which the rarity of the atmosphere in the apparatus undergoing exhaustion can be easily estimated. *d d* is a millimetre-scale with pointed end, attached to the gauge and capable of being raised or lowered so as to make the point just touch the surface of the mercury. *b* is a reservoir of strong sulphuric acid, exposing as much surface as possible, but allowing the air to pass across it without resistance. The mercury joint *c'* may either be closed with a piece of glass rod ground in, or it may have either of the two pieces of apparatus *i* and *k* fitted to it. *k* is a mercurial siphon gauge, which is useful for measuring very high rarefactions in experiments where difference of pressure equal to a tenth of a millimetre of mercury is important. *i* is for still higher rarefactions; it is simply a small tube having platinum wires sealed in, and intended to be attached to an induction-coil. At exhaustions beyond the capabilities of the mercurial gauge I can still get valuable indications of the nearness to a perfect vacuum by the resistance of this tube. I have frequently carried exhaustion to such a point that an induction-spark will not strike across the small distance (¼ inch) separating the wires of the vacuum-tube. *h* is the mercury-tap usually employed for letting air into the apparatus, and also for moistening the interior of the pump with oil of vitriol. *l* is a spiral of glass for attaching the various pieces of apparatus requiring exhaustion. As blown or fused joints are indispensable, this form of connecting piece is adopted to ensure the necessary flexibility. *m* is a trap to catch any air which might leak in through the platinum tap *f*, or the various joints in the lower part of the tubing *g*.

The reservoir A being filled with mercury, the tap I is turned off and the reservoir is raised to the top of the slide where it is supported by the detent T. On opening the tap *f* the mercury rises in the tube *f h*, and, falling through the chamber N, carries with it the air contained in the tube R, and in the apparatus attached to the tube *l*, as in the ordinary Sprengel pump. At N the tubing is enlarged in order that the mercury may not be forced up the tube R, as otherwise frequently happens if the tubes or the mercury gets soiled.

J, J are iron brackets supporting the apparatus. S is a large inverted glass receiver, to collect the small portions of mercury which are unavoidably and constantly being spilled; it should contain a little weak alkaline solution.

The part of the tubing *g*, *f*, *h*, N forms a barometric siphon arrangement, which effectually prevents air getting into the pump from the reservoir A when the mercury has completely run out. In this case no harm whatever is done to the operation: the vacuum is not injured; and the exhaustion proceeds immediately on retransferring the mercury from the reservoir H to the reservoir A, and raising A again into its place. The apparatus, as thus arranged, is readily manageable with certainty of obtaining a barometric vacuum.

The mercury fall-tube of a pump in constant use frequently wants cleaning. I find the most effectual means of doing this is to put oil of vitriol into the funnel *h*, and then, by slightly loosening the glass stopper, allow a little of the strong acid to be carried down the tube with the mercury. With care this can be effected without interfering with the progress of exhaustion. The residual acid adhering to the walls of the chamber N does good rather than harm. When sufficient sulphuric acid has run into the fall-tube, the funnel-stopper can be perfectly closed by pressing it in with a slight twist and then filling up with mercury.

Many physicists have worked on the subject of attraction and repulsion by heat. In 1792 the Rev. A. Bennet recorded the fact that a light substance delicately suspended in air was attracted by warm bodies; this he ascribed to air-currents. When, by means of a lens, light was focused on one end of a delicately suspended arm, either in air or in an exhausted receiver, no motion could be perceived distinguishable from the effects of heat. After Mr. Bennet the subject has been more or less noticed by Laplace, Libri, Fresnel, Saigey, Forbes, Baden Powell, and Faye; but the results have been unsatisfactory and contradictory.

My first experiments were performed with apparatus made on the principle of the balance. An exceedingly fine and light arm is delicately suspended in a glass tube by a double-pointed needle; and at the ends are affixed balls of various materials. Amongst the substances thus experimented on I may mention pith, glass, charcoal, wood, ivory, cork, selenium, platinum, silver, aluminium, magnesium, and various other metals. The beam is usually either of glass or straw.

The apparatus, consisting of a straw beam and pith-ball ends, being fitted up as here shown attached to the pump, and the whole being full of air to begin with, I pass a spirit-lamp across the upper part of the tube just over one of the pith-balls. The ball rises. The same effect is produced when a bulb of hot water or even the warm finger is placed over the pith-ball.

On working the pump and repeating the experiment, the attraction to the hot body gets less and less, until it becomes *nil*, and after a certain barometric pressure is passed, the attraction gives place to repulsion, which gets stronger and stronger as the vacuum approaches perfection.

In order to illustrate more strikingly the influence exerted by a trace of residual air, an apparatus (fig. 2) is here shown in which the source of heat (a platinum spiral, *a*, rendered incandescent by electricity) is inside the glass tube instead of outside it as before. A mass of magnesium, *b*, turned conical, is suspended in a glass tube, *c d e*, by a fine platinum wire of such a length as to vibrate seconds. The upper end of the platinum wire is sealed into the glass at *e*, and passes through to the outside for the purpose of electrical experiments. The platinum spiral is arranged so that when the pendulum hangs free the magnesium mass is about ¼ inch from it. In air the red-hot spiral produces decided attraction on the magnesium; and by properly timing the contacts with the battery, a considerable swing can be accumulated. On perfectly exhausting the apparatus, however, the incandescent spiral is found to energetically repel, and a very few contacts and breaks properly timed are sufficient to get up the full swing the pendulum is capable of.

A simpler form of the apparatus for exhibiting the phenomena of attraction in air and repulsion in a vacuum consists of a long glass tube *a b* (fig. 3) with a globe *c* at one end. A light index of glass with pith-balls at the ends *d*, *e* is suspended in this globe by means of a cocoon fibre. When the apparatus is full of air at ordinary pressure, a ray of heat or light falling on one of the pith-balls gives a movement indicating attraction.

When the apparatus is exhausted until the barometric gauge shows a depression of 12 millims. below the barometer, neither attraction nor repulsion results when radiant light or heat falls on the pith. When the vacuum is as good as the pump will produce, strong repulsion is shown when radiation is allowed to fall on one end of the index. The heat of the hand, or even of the body several feet off, is quite sufficient. The action is in proportion to the surface acted on rather than to the mass.

The barometric position of the neutral point dividing attraction from repulsion varies with the density of the mass on which radiation falls, on the ratio of its mass to its surface, and in a less degree on the intensity of radiation. In the case of pith it is seen to lie at about 12 millims. below a barometric vacuum, whilst with a heavy metal it is within a tenth of a millim. of a vacuum. Experiments to try to determine the law governing the position of the neutral point are now in progress.

Ice, or a cold substance, produces the opposite effects to heat. Thus a bar of pith suspended in a vacuum is energetically repelled by the warm hand, whilst it is as strongly attracted by a piece of ice. Cold being simply negative heat, it is not easy at first sight to understand how it can produce attraction. The law of exchanges, however, explains this perfectly. The pith index and the whole of the surrounding bodies are incessantly exchanging heat-rays; and under ordinary circumstances the income and expenditure of heat are in equilibrium. A piece of ice brought near one end of the index cuts off the influx of heat to it from that side, and therefore allows an excess of heat to fall upon it from the opposite side. Attraction by a cold body is thus seen to be only repulsion by the radiation from the opposite side of the room.

Instruments of the kind just described are perhaps the best for exhibiting large and striking movements of attraction or repulsion. Two glass globes 4 inches in diameter, fitted up with bars of pith $3\frac{1}{2} \times \frac{1}{2}$ inch, are now before you. One is full of air at ordinary pressure, whilst the other is completely exhausted. A touch with a finger on a part of the globe near one extremity of the pith will drive the bar round over 90°, in the vacuum. In air the attraction is not quite so strong.

If I place a lighted candle an inch or two from the vacuous globe, the pith bar commences to oscillate. The swing gradually increases in amplitude until one or two complete revolutions are made. The torsion of the suspending fibre here interferes, and the vibrations proceed in the opposite direction. The movement continues as long as the candle burns. This continued movement ceases if the source of radiation is removed some distance off; the pith index then sets equatorially. The cause of the continued vibration when the radiant body is at a particular distance from the pith is easy to understand on the supposition that the movement is due to the direct impact of waves on the suspended body.

For more accurate experiments I prefer making the apparatus differently. Fig. 4 represents the best form. *a b* is a glass tube, to which is fused at right angles another, narrower tube, *c d*; the vertical tube is slightly contracted at *e* so as to prevent the solid stoper *d*, which just fits the bore of the tube, from falling down. The lower end of the stopper *d e* is drawn out to a point; and to this is cemented a fine glass thread about 0·001 inch diameter, or less, according to the torsion required.

At the lower end of the glass thread an aluminium stirrup and a concave glass mirror are cemented, the stirrup being so arranged that it will hold a beam *f g* having masses of any desired material at the extremities. At *c* in the horizontal tube is a plate-glass window cemented on to the tube. At *b* is also a piece of plate glass cemented on. Exhaustion is effected through a branch tube *h* projecting from the side of the upright tube. This is sealed by fusion to the spiral tube of the pump, The stopper *d e*, and the glass plates *c* and *b*, are well fastened with a cement of resin 8 parts and bee's-wax 3 parts*.

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* This is the best cement I have used for standing a vacuum: for a few hours it is perfect. But at the highest exhaustions it seems to leak in the course of a day or two. India-rubber joints are of no use in these experiments, as, when the vacuum is near upon perfect, they allow oxygenized air to pass through as readily as the pump will remove it. Whenever possible the glass tubes should be united by fusion; and where this is impracticable mercury joints should be used. The best way to make these is to have a well-made perforated conical stopper, cut from plain india-rubber, fitting into the wide funnel-tube of the joint and carrying the narrow tube. Before fitting the tubes in the india-rubber this is heated in a spirit-lamp until its surface is decomposed and very sticky; it is then fitted into its place; mercury is poured over, and oil of vitriol on the top of that. When well made, this joint seems perfect.

The advantage of a glass-thread suspension is that the beam always comes back to its original position. Before you is an instrument of this description, perfectly exhausted and fitted up with pith plates at each extremity. A ray of light from the electric lamp is thrown on to the mirror *c*, and thence reflected on to the opposite wall. The approach of a finger to either extremity of the beam causes the luminous index to travel several feet, showing repulsion. A piece of ice brought near causes the spot of light to travel as much in the opposite direction.

Here is another form of the apparatus (fig. 5). The letters and description are the same as in fig. 4, the apparatus, however, being double. The pieces *f*, *g* on the end of one beam consist of platinum-foil exposing a square centimetre of surface, whilst the extremities f', g' on the other beam consist of pith plates of the same size. It has already been explained that the neutral point of rarefaction for platinum is much higher than for pith; consequently at a pressure intermediate between these two neutral points, radiation ought to cause the platinum to be attracted and the pith to be repelled. This is seen to be the case. A wide beam of radiant heat thrown in the centre of the tube on to the plates g, f' causes g to be attracted and f' to be repelled, as shown by the light reflected from the mirrors c, c'. The atmospheric pressure in the apparatus is equal to about 40 millims. of mercury.

The position of the neutral point not only depends on the density of the body acted on by radiation, as in the above case, but also on the relation of surface to mass. Thus a square centimetre of thin platinum-foil on the extremity of the beam requires a lower exhaustion for neutrality than a thicker piece exposing the same surface. Also a flat disk of platinum has a lower neutral point than the same weight of platinum in the form of a sphere.

Intensity of radiation likewise affects the neutral point. With pith extremities a point of rarefaction can be obtained at which the warm fingers repel and incandescent platinum attracts.

During the course of this lecture I have spoken frequently of repulsion by *heat*, and have used a spirit-lamp as a source of heat to illustrate the facts described. I now wish to show that these results are not confined to the heating rays of the spectrum, but that any ray, from the ultra-red to the ultra-violet, will produce repulsion in a vacuum.

In my own laboratory I have used sunlight, and have experimented with a very pure spectrum, taking precautions to avoid any overlapping or diffusion of one part of the spectrum with another. Here I can only use the electric light, and, in order to get results visible at a distance, the spectrum cannot be very long.

The spectrum is formed with one disulphide-of-carbon prism, and is projected on to the screen by a lens. Immediately behind the screen is an exhausted bulb, having a movable index with pith terminals suspended with a cocoon fibre (fig. 3). This is delicate enough to swing over 90° with a touch of the finger, and it will even move under the influence of a ray of moonlight. I first of all arrange the spectrum so that the extreme red would fall on one pith disk were it not for the screen. On removing the screen the index immediately retreats, making nearly half a revolution.

I now replace the screen, and arrange the spectrum so that the invisible ultra-violet rays are in a position to fall on the pith disk. On removing the screen the index at once behaves as it did under the influence of the red rays, and is driven away twenty or thirty degrees. The action is not so powerful as when the other end of the spectrum is used; but this may partly, if not wholly, be accounted for by the much greater concentration of energy at the red end of the spectrum, and expansion at the violet end, when using glass or disulphide-of-carbon prisms.

I now, without disturbing the position of the spectrum, interpose in the path of the rays a cell containing a solution of iodine in disulphide of carbon, which is opaque to the luminous and ultra-violet rays, but transparent to the invisible heat-rays. Not a trace of repulsion is produced. The iodine solution is now removed and the ultra-violet rays again fall on the pith, producing strong repulsion. A thick screen of clear alum cut from one of Mr. Spence's gigantic crystals is now interposed; but no effect whatever is produced by it, the ultra-violet rays acting with unabated energy. As alum cuts off all the dark heat-rays, this experiment and the one before it prove the sufficient purity of my spectrum.

The spectrum is again turned until the dark ultra-red heating rays fall on the pith. The movement of repulsion is energetic. The iodine solution, interposed, cuts off apparently none of the action. The alum plate cuts off a considerable amount, but by no means all. On uniting the alum and the iodine solution the whole of the spectrum is obliterated, and no action is produced, whatever be the ray which would, were it not for this double sifting, fall on the pith.

Throughout the course of these investigations, which have occupied much of my spare time for some years, I have endeavoured to keep in my mind the possible explanations which may be given of the actions observed; and I have always tried, by selecting some circumstances and excluding others, to put each hypothesis to the test of experiment.

The most obvious explanation is, that the movements are due to the currents formed in the residual gas which theoretically must be present to some extent even in those vacua which are most nearly absolute.

Another explanation is, that the movements are due to electricity developed on the moving body or on the glass apparatus by the incident radiation.

A third explanation has been put forward by Professor Osborne Reynolds, in a paper which was read before the Royal Society on June 18th last. He considers the results to be due to evaporation and condensation.

I will discuss these explanations in order.

First, the air-current theory. However strong may be the reasons in favour of this explanation, they are, I think, answered irrefragably by the phenomena themselves. It is most difficult to believe that the residual air in a Sprengel vacuum, when the gauge and barometer are level, can exert, when gently warmed by the finger, an upward force capable of instantly overcoming the inertia of a mass of matter weighing 20 or 30 grains. It must be remembered that the upward current supposed to do this is simply due to the diminished weight of a portion of the gas caused by its increase in volume by the heat applied.

An air-current produced by heat may possibly cause the beam of a balance to rise, may drive a suspended index sideways, and by a liberal assumption of eddies and reflections, may perhaps be imagined to cause these movements to take place sometimes in the opposite directions; but as rarefaction proceeds these actions must certainly get less, and they will cease to be appreciable some time before a vacuum is attained: a point of no action or neutrality will be reached. But this neutral point should certainly be nearer to a vacuum when a light body like pith, exposing much surface, is under experiment, than when the mass acted on is heavy like brass; whereas in practice the contrary obtains. Pith ceases to move under the influence of radiation at a rarefaction of about 7 to 12 millims., whilst brass only ceases to be affected when the gauge and barometer are appreciably level.

But even could the phenomena up to the neutral point be explained by air-currents, these are manifestly powerless to act after this critical point is passed. If a current of air within 7 millims. of a vacuum cannot move a piece of pith, certainly the residual air in a Sprengel vacuum should not have more power; and *à fortiori* the residual gas in a perfect chemical vacuum cannot possibly move a mass of platinum.

It is, however, abundantly demonstrated that, in all cases after this critical point is reached, the repulsion by radiation is most apparent; it increases in energy as the vacuum approaches perfection, and attains its maximum when there is no air whatever present, or at all events not sufficient to permit the passage of an induction-spark.

I will now refer to the electrical explanation. Very early in my investigation, phenomena were noticed which caused me to think that electricity played a chief part in causing the movements. When a hot glass rod is held motionless against the side of an exhausted tube containing a pith index, repulsion takes place in a perfectly regular manner; but if the glass rod has been passed once or twice through the fingers, or is rubbed a few times sideways against the exhausted bulb, the index immediately moves about in a very irregular manner, sometimes being repelled from, and at others attracted to, the side of the glass, where it adheres until the electrical excitement subsides. Friction with the finger produces the same results; and a small spirit-flame causes similar, but much fainter, electrical effects. I soon ascertained, however, that, although electricity is capable of producing many movements similar to those caused by radiation, they are never so alike as to be mistaken. Electricity frequently interferes with, disturbs, or neutralizes the true action of radiation; but it acts in such a manner as to show that it is not the primary cause of the movement. At the highest rarefactions, and when special precautions have been taken to avoid the presence of aqueous vapour, slight friction with the finger against the bulb, or a touch with the flame of a spirit-lamp, excites so much electrical disturbance in the pith and other indexes that accurate observations become impossible with them for several hours. I have tried many means of neutralizing the electrical disturbance; but they are only partially successful, and at the highest rarefactions interference through electrification is very troublesome.

I may draw attention to the following experiments, which are devised with the object of showing that the attractions and repulsions are not due to electricity.

In describing the pendulum apparatus (fig. 2) which I set in motion at the early part of this lecture, I explained that the mass of magnesium forming the weight was in metallic contact with the platinum wire which supported it, and that the upper end of this platinum wire was fused into the glass tube and passed through to the outside. With this apparatus I have tried a great number of experiments. I have connected the projecting end of the platinum wire with "earth," with either pole of an induction-coil the other being insulated more or less, with either pole of a voltaic battery, with a delicate electroscope; I have charged it with an electrophorus, and have submitted it to the most varied electrical conditions; and still, on allowing radiation to fall upon the suspended mass, I invariably obtain attraction when air is present, and repulsion in a vacuum. The heat has been applied both from the outside, so as to pass through the glass, and also inside by means of the ignited platinum wire; and the results have shown no difference in kind, but only in degree, under electrical excitement. I have obtained interference with the usual phenomena, but never of such a character as would lead me to imagine that the normal results were due to electricity.

It occurred to me that the repulsion might be due to a development of electricity on the inner surface of the glass bulb or tube under the influence of the radiation as it passed from the glass into the vacuum. This appears to be disproved by the fact that the results are exactly the same whether the radiation passes through the glass, or whether it is developed inside the apparatus as in the above instance.

I have produced exactly the same phenomena whether the exhausted apparatus has been standing insulated in the air, or whether it was completely immersed in water connected electrically with "earth," or surrounded with wet blotting-paper.

Here are two experiments which bear on this subject. A straw beam furnished with brass balls at each end is suspended on a double-pointed needle, and the brass balls and needle are placed in metallic connexion by means of fine platinum wire. The needle does not rest on the sides of the glass tube, but in steel cups, to which is soldered a platinum wire passing through the glass tube and connected with "earth." The tube is then exhausted, and the usual experiments are tried with hot and cold bodies, both with and without a wet blotting-paper cover. In all cases the moving beam behaves normally, being repelled by heat and attracted by cold.

An apparatus is prepared similar to that shown in fig. 4. The inside of the tube *a b* is lined with a cylinder of copper gauze, having holes cut in the centre for the passage of the supporting thread *d c* and the index ray of light falling on the mirror *c*, and holes at each end to admit of the plates *f* and *g* being experimented with. A wire attached to the copper gauze passes through a hole in the plate *b*, so as to give me electrical access to the copper gauze lining. Under the most diverse electrical conditions, whether insulated or connected with "earth," this apparatus behaves normally when exhausted.

A further reason why electricity is not the cause of the movements I have described is, that they are not only produced by heat, but also by ice and cold bodies. Moreover I shall presently show that any ray of the spectrum, besides those red and ultra-red rays which produce dilatation of mercury in a thermometer, excite an electric current between antimony and bismuth couples, and cause a sensation of warmth when falling on the skin, will produce the effect of repulsion in a vacuum. It is therefore to my mind abundantly proved that electricity, such as we at present know this force, is not a chief agent in these attractions and repulsions, however much it may sometimes interfere with and complicate the phenomena.

I will now discuss Professor Osborne Reynolds's theory, that the effects are the results of evaporation and condensation. In my exhausted tubes he assumes the presence of aqueous vapour, and then argues as follows:—"When the radiated heat from the lamp falls on the pith, its temperature will rise, and any moisture on it will begin to evaporate and to drive the pith from the lamp. The evaporation will be greatest on that ball which is nearest to the lamp; therefore this ball will be driven away until the force on the other becomes equal, after which the balls will come to rest, unless momentum carries them further. On the other hand, when a piece of ice is brought near, the temperature of the pith will be reduced, and it will condense the vapour and be drawn towards the ice."

Professor Reynolds has tried an experiment with pith-balls attached to a light stem of glass and suspended by a silk thread in a glass flask. The exhaustion was obtained by boiling water in the flask and then corking it up and allowing it to cool. The gauge showed an exhaustion of from ½ to ¾ of an inch. The pith-balls behaved exactly as I have already shown they do at that degree of exhaustion, heat repelling and ice attracting. He found that the neutral point varied according to whether air was present with the aqueous vapour, or whether the vapour was pure water-gas. Professor Reynolds states:—"From these last two facts it appears as though a certain amount of moisture on the balls was necessary to render them sensitive to the heat. . . . . These experiments appear to show that evaporation from a surface is attended with a force tending to drive the surface back, and condensation with a force tending to draw the surface forward."

It does not appear that Professor Reynolds has tried more than a few experiments; and he admits that they were in reality undertaken to verify the explanation above quoted. I have worked experimentally on this subject for some years; and the last experiment recorded in my notebook is numbered 584. From the abundant data at my disposal, I can find many facts which will, I think, convince you that this hypothesis has been arrived at on insufficient evidence.

In the first place, I will show that the presence of moisture or of a condensable vapour is not necessary. Besides pith, which from its texture and lightness might be supposed to absorb and condense considerable quantities of vapour, I have used glass, mica, and various metals; and with a proper amount of exhaustion they all act in the same manner. The fact that the neutral point for platinum is close upon a vacuum, whilst that for pith is so much lower, tends to show that the repulsion is not due to any recoil caused by condensable vapour leaving the surface under the influence of heat. Were it so, it would certainly require more vapour to be present when platinum had to be driven backwards than when pith had to be moved; but the contrary obtains in all cases. The rule seems to be, the greater the density the higher the neutral point.

I have worked with all kinds of vacua; that is to say, I have started with the apparatus filled with various vapours and gases (air, carbonic acid, water, iodine, hydrogen, &c.); and at the proper rarefaction I find no difference in the results which can be traced to the residual vapour. A hydrogen vacuum seems neither more nor less favourable to the phenomena than does a water vacuum, or an iodine vacuum.

If moisture be present to begin with, I find it necessary to allow the vapour to be absorbed by the sulphuric acid of the pump, and to continue the exhaustion, with repeated heating of the apparatus, until the aqueous vapour is removed. Then and then only do I get the best results.

When pith is employed as the index, it is necessary to have it thoroughly dried over sulphuric acid before using it, and during the exhaustion to keep it constantly heated to a little below its charring-point, in order to get the greatest sensitiveness.

Professor Reynolds says, "In order that these results might be obtained, it was necessary that the vapour should be free from air." On the contrary, I find the results take place with the greatest sharpness and rapidity if the residual gas consists of nothing but air or hydrogen.

Professor Reynolds further says, "Mr. Crookes only obtained his results when his vacuum was nearly as perfect as the Sprengel pump would make it. Up to this point he had nothing but the inverse effects, viz. attraction with heat and repulsion with cold." In the abstract of my paper published in the Proceedings of the Royal Society, I describe an experiment with a pith-ball apparatus in which the neutral point is 7 millims. (about $\frac{1}{4}$ inch) below the vacuum, repulsion by heat taking place at higher exhaustions. At the Royal Society *Soirée*, April 22, 1874, I showed, and fully described in print, the apparatus now before you, consisting of a pith bar suspended by a cocoon fibre in a glass bulb, from which the air is exhausted until the barometric gauge shows a depression of 12 millims. below the barometer. Neither attraction nor repulsion results when radiant light or heat falls on the pith. Exhaustions of 7 and 12 millims. are certainly very inferior vacua for a Sprengel pump.

As a matter of fact, however, I have obtained repulsion by radiation at far higher pressures than these. The true effect of radiation appears to be one of repulsion at any pressure, overbalanced when a gas is present by some cause—possibly air-currents, but probably not. I have already explained that the barometric height of this neutral point dividing attraction from repulsion varies with the density of the substance on which radiation falls, on the relation which the mass bears to the surface, and on the intensity of radiation. By modifying the conditions it is not difficult to get repulsion by radiation when the apparatus is full of air at nearly the normal pressure of the atmosphere.

Professor Reynolds again says, "The reason why Mr. Crookes did not obtain the same results with a less perfect vacuum was because he had then too large a proportion of air, or non-condensing gas, mixed with the vapour." On this I may remark that the writer, before he explained how it was I could *not* get certain results, should have made sure that what he assumed to be the case was really so. I have not the least difficulty in showing repulsion by heat in imperfect vacua when mixed vapours and gases are present.

In my arguments against the air-current theory, I have shown that the best results are obtained when the vacuum is so nearly perfect that an induction-spark will not pass through it. This is an equally good argument against the presence of a condensable vapour as it is against that of air.

From the construction of my Sprengel pump I am satisfied that the vapour of mercury is absent from the apparatus.

The following experiments have been specially tried with the object of testing this theory. A tolerably thick and strong bulb is blown at the end of a piece of combustion-tubing; and in it is supported a bar of aluminium at the end of a long platinum wire. The whole is attached to the Sprengel pump, and exhaustion is kept going on for about two days, until a spark will not pass through the vacuum. During this time the bulb and its contents are frequently raised to an incipient red heat. At the end of that time the tube is sealed off, and the bar of aluminium is found to behave exactly as it would in a less perfectly exhausted apparatus; viz. it is repelled by heat. A similar experiment, attended with similar results, has been tried with a glass index. It is impossible to conceive that in these experiments sufficient condensable gas was present to produce the effects Professor Reynolds ascribes to it. After the repeated heatings to redness at the highest attainable exhaustion (the gauge and the barometer being level for nearly the whole of the 48 hours), it is impossible that sufficient vapour or gas should condense on the movable index to be instantly driven off, by the warmth of the finger, with recoil enough to drive backwards a heavy piece of metal.

My own impression is that the repulsion accompanying radiation is directly due to the impact of the waves upon the surface of the moving mass, and not secondarily through the intervention of air-currents, electricity, or evaporation and condensation. Whether the ætherial waves actually strike the substance moved, or whether at that mysterious boundary-surface separating solid from gaseous matter there are intermediary layers of condensed gas which, taking up the blow, pass it on to the layer beneath, are problems the solution of which must be left to further research.

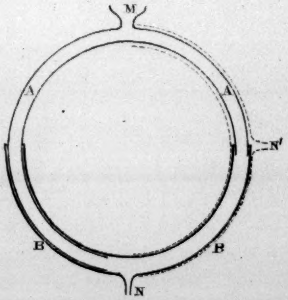
In giving what I conceive to be reasonable arguments against the three theories which have been supposed to explain these repulsions, I do not wish to insist upon any theory of my own to take their place. The one I advance is to my mind the most reasonable, and as such is useful as a working hypothesis, if the mind must have a theory to rest upon. *Any* theory will account for *some* facts; but only the true explanation will satisfy *all* the conditions of the problem, and this cannot be said of either of the theories I have already discussed.

My object at present is to ascertain facts, varying the conditions of each experiment so as to find out what are the necessary and what the accidental accompaniments of the phenomena. By working steadily in this manner, letting each group of experiments point out the direction for the next group, and following up as closely as possible, not only the main line of research, but also the little bylanes which often lead to the most valuable results, after a time the facts will group themselves together and tell their own tale. The conditions under which the phenomena invariably occur will give the laws; and the theory will follow without much difficulty. To use the eloquent language of Sir Humphry Davy, "When I consider the variety of theories which may be formed on the slender foundation of one or two facts, I am convinced that it is the business of the true philosopher to avoid them all together. It is more laborious to accumulate facts than to reason concerning them; but one good experiment is of more value than the ingenuity of a brain like Newton's."

XI. *Modification of the usual Trombone Apparatus for showing the Interference of Sound-bearing Waves. By* W. F. Barrett, *F.R.S.E. &c., Professor of Physics in the Royal College of Science, Dublin.*

A simple apparatus for showing the interference of sound-bearing waves may be made by employing a circular arrangement of tubes, one sliding within the other. One tube, A, to which the mouthpiece M is fixed, is three fourths of a circle; the other tube, B, to which the nozzle N is attached, is half a circle, and of such diameter that it slides freely over the tube A.

When the nozzle is diametrically opposite the mouthpiece, the path of the sound-waves is of equal length, and hence the sound from any convenient source placed near to or within the mouthpiece is distinctly heard. By turning the nozzle towards N′, in the direction shown by the dotted lines, one limb of the tube is lengthened whilst the other is correspondingly shortened; the path of the waves being now unequal, a point is soon reached where the sound is nearly obliterated.



Employing a suitable source of sound, and a sensitive flame or a resonant jar as a phonoscope, an audience can perceive at once the gradual destruction of the sonorous pulses; and moreover the relative lengths of the two branches of the tube clearly indicate the principle of interference thus illustrated.

One instrument I made was 2 feet in diameter, of 1-inch-square zinc tubing; another and better instrument (skilfully made by Mr. R. H. Ridout) was of brass tubing, 1 foot in diameter, the one limb being ½-inch, the other ⅝-inch tube. About 18 inches in diameter would probably be the best and most convenient size. In making the experiment, care should be taken to avoid (α) the conduction of sound to the ear by the metal substance of the instrument; (β) the direct transmission of sound through the surrounding air. The latter can be overcome by attaching a sufficiently long gutta-percha tube to M, thus removing the mouthpiece to a distance from the ear. The former can be obviated to some extent by having an inelastic mouthpiece or similar covering to the end of the tube. But Mr. Woodward's device of putting a source of sound, such as a reed, entirely within the tube, and a trumpet mouthpiece at N, is undoubtedly the best and most suitable class method of making the experiment.

P.S.—With an ordinary pitch-pipe inserted at N, I have to-day (July 25) repeated the experiment to the class of science teachers now at South Kensington. A continuous blast of air was driven through the pipe from an acoustic bellows; and the loud note heard at first was *utterly extinguished* by altering the relative lengths of the tubes. By pushing the tube still further round the note again came out; thus the sound of the pitch-pipe could be turned on and off at pleasure. Extinction is not confined to a mere line in adjusting the pipe, but spreads over a short and definite range. In this case it is probably, as Professor Goodeve suggests, the interference of two resonant columns of air, rather than the coalescence of two progressive waves in opposite phases.

PROCEEDINGS

OF

THE PHYSICAL SOCIETY

OF LONDON.

November 7, 1874.

XII. *On Salt Solutions and Attached Water.*

By FREDERICK GUTHRIE.

§ 1. THE interesting paper read to the Physical Society of London by Dr. J. Rae, the arctic explorer, on the comparative saltness of freshly formed and older ice-floes*, induced me to examine some of the physical properties of brine and some other of the aqueous solutions of the salts of the sea. This examination has extended itself to some other salts; for before attacking the problem of the freezing of sea-water, the composition of which may be little less complex than that of the earth itself, it seemed advisable to enrich our knowledge of the properties of the solutions of some of its more abundant constituents in the separate form. Some of the experimental results unexpectedly opened so wide and fascinating a field of inquiry, that I have been compelled to make some sort of survey of it; and though the results there gathered are of sufficient interest to be considered by themselves as physical facts, they will assuredly also be concerned in the establishment of a sufficient theory of thalattology.

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* Proc. Phys. Soc. May 9, 1874. Phil. Mag. July 1874, p. 56.

Chloride of Sodium.

§ 2. Being fortunately in possession of a large quantity of extremely pure rock-salt in fine crystals, I satisfied myself by the spectroscope of the absence of potassium and lithium. By the usual tests, magnesium and sulphuric acid were also shown to be absent. A determination of the chlorine agreed so closely with the theoretical quantity that not more than 0·2 per cent. of any metal having an atomic weight differing by more than 5 from that of sodium could be present, supposing it to be a chloride. On converting the chloride into a sulphate an equally accordant result was got. This convinced me that I had a perfectly pure substance.

§ 3. *Preparation of the Brine.*—The clearest lumps of the rock-salt were washed in common and then in distilled water. With these a large beaker was filled. Distilled water was poured on so as to fill the beaker about two thirds. If part of the salt is thus always above the surface of the brine, the latter appears to be saturated in twenty-four hours. After forty-eight hours the brine was poured into another beaker and allowed to stand until crystals began to be formed. It was then transferred to a stoppered bottle and was ready for use. Although Regnault and others have shown that for temperatures above 0° C. the solubility of chloride of sodium in water is constant, yet the precaution was taken of suspending a muslin bag containing some crystals of salt from a string passing through the cork of the bottle so that the crystals touched the surface of the brine. By this means the perfect saturation of the brine was assured. At 18° C. this solution contained 26·2724 per cent. of salt. The solution was weighed in a long-necked flask, and was evaporated to dryness and heated to about 300° C.

The following numbers show the results obtained by some other experimenters:—

				per cent.	spec. grav.
Kopp . .	Sat. at	25° C.	contains	26·32	
Kastner .	„	18·75	„	26·72	1·2046
F. G. . .	„	18	„	26·27	
F. G. . .	„	21			1·1997
F. G. · .	„	26			1·2011

The above specific gravities are referred to water at the same temperature as the brine, as far as my own experiments are concerned.

§ 4. *Cold produced on mixing saturated Brine with Water.*—To examine this point, a series of covered beaker-glasses containing known weights of water, and a similar series containing saturated brine, were placed side by side in a tin tray containing water and resting on flannel. The whole arrangement was covered with flannel and allowed to stand over night. In the morning the temperature of eight or ten was found to be exactly 21° C. The liquids were prepared by measuring out from a burette in proportions that, taking 1·1997 as the specific gravity of the brine, the ratios by weight were as in columns 3 and 4 (Table I.). The experiments were performed by taking out a water-beaker, rapidly drying its outside, pouring in the brine, stirring with the thermometer, and at once reading.

TABLE I.

Cold produced on mixing saturated Brine with various proportions of Water by weight. Initial temperature 21° C.

(1) Water, in grams.	(2) Sat. brine, in grams.	(3) Water, per cent.	(4) NaCl, per cent.	(5) Fall of temperature.
				° Cent.
60	140	81·611	18·389	0·4
65	135	82·268	17·732	0·5
70	130	82·925	17·075	0·6
75	125	84·081	15·919	0·75
80	120	84·238	15·762	0·8
85	115	84·895	15·105	0·85
90	110	85·552	14·448	0·9
95	105	86·198	13·802	0·85
100	100	86·870	13·130	0·85
105	95	87·522	12·478	0·85
110	90	88·289	11·821	0·85
115	85	88·835	11·165	0·9
120	80	89·492	10·508	0·9
125	75	90·149	9·851	0·9
130	70	90·806	9·194	0·9
135	65	91·463	8·537	0·75
140	60	92·119	7·881	0·65
145	55	92·776	7·224	0·6
150	50	93·433	6·567	0·6
155	45	94·089	5·911	0·6
160	40	94·746	5·254	0·5

The greatest depression of temperature accordingly in the range of ratio between 84 and 90 of water and 16 and 10 of salt, and the greatest depression of temperature to be got by mixing saturated brine with water, is under no circumstance quite 1° C.

§ 5. If now anhydrous salt on dissolving in water absorbs heat only by reason of its conversion into a liquid, and if no hydrate of salt exists at the ordinary temperature above 0° C., no such absorption of heat as we have seen to occur could ensue on mixing brine and water. Two series of experiments are therefore at once demanded :—the first to see what proportion must exist between anhydrous salt and water to absorb the most heat; the second to see if at any ratio between salt and water an abnormal change of density takes place.

§ 6. *Cold produced on mixing* NaCl *with Water.*—A quantity of rock-salt having been finely powdered, was passed through muslin so as to be quite mealy. Various quantities of distilled water were weighed into beakers, and various quantities of the rock-salt were thrown in under constant stirring. The abasement of temperature being noted gave the results of Table II. The temperature of the salt was about 2° C. above that of the water. And it is seen that the greatest abasement of temperature occurs when there is about 40–60 of salt to 160–140 of water (say 25 per cent. of salt). This result is what we should expect, remembering that the saturated solution contains 26·27 per cent. of salt.

TABLE II.—Maximum Cold produced on mixing Salt and Water in different proportions by weight.

Temp. of salt = 20°·2 C.

Water, in grams.	NaCl, in grams.	Temp. of water.	Temp. of mixture.	Fall.
190	10	19°·1	18°·0	1°·0
180	20	18·4	17·0	1·4
160	40	18·4	16·4	2·0
140	60	18·4	16·3	2·1
120	80	18·4	16·5	1·9
100	100	18·5	16·7	1·8

§ 7. *Specific Gravity of Brines of various strengths.*—The question as to the source of cold when brines are diluted can be conveniently approached from an examination of the density of brines of different strengths, and the comparison between the observed specific gravities and the theoretical specific gravities, supposing no change of volume to take place. The brine was found to have a specific gravity of 1·2011 at 26° C., the temperature at which the determinations were made. In Table III. the columns 1 and 2 give the weights in grams of the quantities of water and saturated brine. Columns 3 and 4 give the percentage of the water and NaCl in the resulting brines. Column 5 gives the specific gravity found, and column 6 gives the specific gravity calculated under the hypothesis that no alteration of volume ensues.

Table III.—Specific Gravity of Brines of various strengths.

(1) Water, in grams.	(2) Saturated brine, in grams.	(3) Water, per cent.	(4) NaCl, per cent.	(5) Observed spec. grav.	(6) Calculated spec. grav.
0	∞	73·73	26·27	1·2011	
60	140	81·61	18·39	1·1087	1·1328
70	130	82·92	17·07	1·0963	1·1211
80	120	84·24	15·76	1·0874	1·1117
90	110	85·55	14·45	1·0800	1·1014
100	100	86·87	13·13	1·0725	1·0914
110	90	88·29	11·82	1·0650	1·0815
120	80	89·49	10·51	1·0574	1·0718
130	70	90·81	9·19	1·0501	1·0621
140	60	92·12	7·88	1·0428	1·0530
150	50	93·43	6·57	1·0357	1·0434
160	40	94·75	5·25	1·0284	1·0347
170	30	97·06	3·94	1·0246	1·0258

There is accordingly an increase of volume when a solution of salt is diluted; and this is of course connected with the absorption of heat examined in § 4, Table I. When the original temperatures are restored, a mixture of strong brine and water may have a volume two hundredths greater than the sum of the volumes of its constituents. And when the brine contains as little as 4 per cent. of salt, its specific gravity is sensibly smaller than if such a proportion had been the result of the mixture without contraction of the strongest brine and water.

§ 8. *Refraction of Light by Brines of different strengths.*—It seemed, from a consideration of the numbers in Table III., that at or near the point of saturation a definite hydrate of salt exists;

and that this is not merely diluted, but also actually decomposed on the addition of water, so that expansion takes place. This I imagined might be tested by the change in the refractive index of the brine. Accordingly brines of various strengths were placed in a hollow prism of 60°. The refraction was measured on a goniometer (Babinet's) table provided with telescope and collimator. An alcohol-flame containing sodium was employed as a source of light shining through a very fine slit. The minimum refraction being obtained, the D line was split by the spider-thread. In Table IV. the empty prism is considered as giving a displacement of 0°. The angular displacements are alone here given, as they perfectly suffice for the detection of singular values.

TABLE IV.

Refraction of Light by Brines of various strengths.

Temp. =22° C.

Per cent. of brine in solution.	Per cent. of salt in solution.	Displacement.	
∞	26·27	27° 0′ 45″	or 27·0125
70	18·39	26 2 30	„ 26·0416
65	17·07	25 44 0	„ 25·7333
60	15·76	25 32 30	„ 25·5416
55	14·45	25 20 0	„ 25·3333
50	13·13	25 10 0	„ 25·1666
45	11·82	24 59 0	„ 24·9833
40	10·51	24 48 0	„ 24·8000
35	9·19	24 37 30	„ 24·6250
30	7·88	24 26 45	„ 24·4458
25	6·57	24 16 30	„ 24·2750
20	5·25	24 6 0	„ 24·1000
0	0·00	23 23 0	„ 23·3833

These numbers show a singular value at about the 10- to 11-per-cent. solutions.

§ 9. Another method of attacking the question is offered by the examination of the boiling-points of various brines. Accordingly I took solutions made by mixing saturated brine with water, and waiting for a day on which the barometer stood nearly at the mean, I determined the boiling-points of the brines. The vessels were tall copper cylinders. In the Table V. the temperature of the steam as well as that of the brine itself is given. The boiling was, of course, only continued a short time to avoid the error of strengthening the brine.

TABLE V.

Boiling-points of Brines of various strengths in copper cylinder.

Per cent. of NaCl.	Boiling-point in liquid.	Boiling-point in vapour.
26·27 (sat.)	108°·8	107°·0
18·389	104·7	104·2
17·075	104·2	103·1
15·762	104·0	102·6
14·448	103·4	102·5
13·130	103·0	102·3
11·821	102·6	101·9
10·508	102·4	102·1
9·194	102·0	101·7
7·881	101·7	101·3
6·567	101·2	101·0
5·254	101·0	101·0
0·00 (dist. water)	100·4	100·0

We have here again a singular value about the 10- to 11-per-cent. solutions.

§ 10. *Freezing-points of Brines of various strengths.*—The molecular separation of water from salt when a brine is boiling has to some extent a counterpart in the separation of ice from brine when the latter is subjected to cold. If a weak brine such as 9 of water to 1 of saturated brine (that is, 1 of salt to 24·2 of water) is subjected to cold, pure ice begins to be formed in this case at −1°·5. The temperature gradually sinks; but, as has been shown, the solid part consists of ice, which may be completely freed from salt by mere pressure. This formation of ice continues, and the temperature sinks until the inferior limit attainable by an ice-salt freezing-mixture is reached (−22° C.). The brine, of course, becomes richer and richer in salt. In other words, brines richer in salt yield up ice at lower temperatures. Thus, in Table VI. The same brines are examined as were examined in the preceding Tables. The various brines were examined in succession, being contained in small beaker-glasses in an ice-salt freezing-mixture. It is noteworthy that all the brines are supersaturable in regard to ice, so that the temperatures are most exactly determined by two observations. A good deal of ice is allowed to form, and this is then suffered nearly entirely to disappear; the temperature is observed at which the ice begins to increase in quantity when the brine is again subjected to cold.

TABLE VI.

Temperatures at which Brines of various strengths give up ice.

(1) Water, in grams.	(2) Brine, in grams.	(3) Per cent. of NaCl.	(4) Temperature at which ice is first formed.
60	140	18·389	−15°·4
70	130	17·075	−15·0
80	120	15·762	−12·4
90	110	14·448	−11·1
100	100	13·130	− 9·4
110	90	11·821	− 7·7
120	80	10·508	− 7·7
130	70	9·194	− 6·7
140	60	7·881	− 5·4
150	50	6·567	− 4·1
160	40	5·254	− 3·4
180	20	2·627	− 2·1
185	15	1·970	− 1·5
190	10	1·313	− 1·9 (?)
195	5	0·656	− 1·5
200	0	0·000	0·0

Again the brines which contain 10 to 11 per cent. of salt have singular behaviour in regard to the temperature at which they yield ice. These are the very solutions, it will be remembered, which behave singularly in respect of their refractive indices and also of their boiling-points. On comparing with Table III., this singularity does not manifest itself in regard to the specific gravities.

§ 11. *Effect of cooling saturated Brine.*—It is seen from Table VI. that, as far as the strengths of brine there examined extend, the stronger the brine the greater the cold required to separate ice from it. The strongest brine there examined contains 18·389 per cent. of NaCl. If saturated brine (containing 26·27 per cent. of NaCl) be cooled, quite a different class of phenomena ensues. Down to 0° C. no solidification whatever ensues either of ice or of salt. At −7° crystals of the bihydrate are observed to fall ($NaCl+2H_2O$). These present a beautiful appearance of iridescent scales heavier than the mother-brine.

Their composition has been examined by Löwitz, Fuchs, Nölle, and Mitscherlich, whose analyses, however, are not in good accord, that of Löwitz being as much as 10 per cent. different from the calculated percentage of the bihydrate. Ehrenberg and Frankenheim state that the same hydrate is produced when a dilute solution of chloride of sodium evaporates at 15° C., and that it suddenly converts itself into the cubical anhydrous salt and water. At −22° C. the whole of this hydrate appears to be removed. The gradual impoverishment of the mother-brine was tested by maintaining a brine, at first saturated, for half an hour at the successive temperatures −10°, −16°, and −21 to −22° C. The brine had been previously kept at 0° for an hour. The original brine contained 26·2724 of NaCl per cent. It was the mother-brine from each crop of crystals which was subjected to the lower temperature.

		per cent.	
Saturated brine at 0° . . .	contained	26·2724	of NaCl.
Mother-liquor after keeping at −10°	„	24·6528	„
„ „ −10°	„	24·6187	„
„ „ −16°	„	24·1182	„
„ „ −21° to 22°		23·8874	„

It is clear, therefore, that down to −21° the solidification impoverishes the brine,—a result which might be due to the formation of the bihydrate or the precipitation of the anhydrous salt, but which is inconsistent with the formation of ice alone. The determinations were made by weighing 10 or 12 grammes of the brines, when at the atmospheric temperature, into long-necked flasks, evaporating and heating to about 300° C.

§ 12. A quantity of brine which had been thus impoverished by being kept for an hour at −21° to −22° was decanted into another flask and further cooled by contact with solid carbonic acid and ether. The whole (3 or 4 ounces) solidified; and the temperature remained perfectly constant at −22° to −23° (say −23°) until the last drop had frozen at −23°; it then sank rapidly.

If the vessel be continually shaken during crystallization, the form of the crystals may be very clearly seen. While the anhydrous salt crystallizes in the well-known cubes, the bihydrate separates as iridescent scales, and the body we are now considering solidifies in acicular bundles radiating from nuclei, and much resembling in appearance the supersaturated solution of sulphate of sodium when solidifying. It is, however, of a more than pearly whiteness, and finally of complete opacity.

§ 13. To see whether we have here indeed a definite hydrate, the whole was remelted by the warmth of the hand and successively partially refrozen under continual agitation. Each mother-liquor was poured off into a weighed flask and partially solidified, and so on five times in succession. The solid residues were then allowed to melt and get to the temperature of the air; they were then evaporated in the usual way. The portion which remained to the last was not frozen, in order to see whether its composition was the same as that of the parts previously removed by solidification.

Table VII.

Percentage of salt in fractionally solidified NaCl brine below 21°.

Temperature of solidification.		per cent.	
−21° to −22°,	contained	23·7232	of NaCl.
−22	„	23·6581	„
−22	„	23·7262	„
−23	„	23·8201	„
−23	„	23·3431	„
−23	„	23·3478	„

The nearest molecular relationship indicated by these numbers is

$$2\,NaCl + 21\,H_2O.$$

The formula	$NaCl + 9\,H_2O$	requires	26·5	per cent.	of salt.
„	$NaCl + 10\,H_2O$	„	24·5	„	„
„	$NaCl + 11\,H_2O$	„	22·4	„	„
„	$2\,NaCl + 21\,H_2O$	„	23·6	„	„

§ 14. In these the salient point is the composition of the final mother-liquor, which is essentially the same as that of the successively separated solids.

Accordingly a salt-ice freezing-mixture is just capable of impoverishing saturated brine by withdrawal of salt-rich ingredients (namely the bihydrate) to such an extent that the unsolidified part is homogeneous, in the sense of being solidifiable as a whole. And such solidification takes place immediately below the temperature −21° to −22°, which is the lowest temperature to be got by an ice-salt freezing-mixture. I presume that if the two solids, ice and salt, could be presented to one another in a state of indefinitely fine division, this proportion of 23·6 of salt to 76·4 of ice would act most promptly and continuously as a freezing-mixture, because the formation of the bihydrate would not then occur.

I am disposed to think that the hydrate of salt, the genesis of which is here described, may have the composition

$$NaCl+10H_2O;$$

for under the circumstances of its formation and analysis I only see one serious source of error; and that is the condensation of moisture from the air upon the surface of the cold brine. This would tend to make the amount of salt found too small. To this point we shall have to return.

§ 15. *Physical Condition of a Freezing-mixture.*—The fact that the *rationale* of a freezing-mixture is the liquefaction of solids seems to demand at once the conclusion that a freezing-mixture (of say ice and salt) must always be partially liquid.

Nevertheless, if a mixture be made of about three parts by weight of finely powdered rock-salt and one part of finely crushed ice, the liquefaction which ensues is followed by an apparent regelation so complete that the whole can be handled as a solid mass, and becomes indeed perfectly dry. The cause of this is, I have no doubt, due to the temporary existence of a supersaturated solution of $NaCl+2H_2O$, a body which is formed at about −10°, Löwitz (−5°, Nölle) (from −3° to −20° continuously, F. G.), when saturated brine is artificially cooled. It is, of course, impracticable to separate this substance from the general mass of the freezing-mixture with sufficient precision to allow of its analysis. The phenomenon of solidification, moreover, only lasts a few minutes. Liquefaction ensues, and the temperature, which had made a pause, again sinks rapidly.

§ 16. The minimum temperature of a salt-ice freezing-mixture seems to be attained between the somewhat wide margins of 3 of salt to 1 of ice, and 1 of salt to 2 of ice. The lowest temperature appears to be −21° to −22° C.

§ 17. *Liquid portion of Freezing-mixture.*—It is clear that the liquid portion of a freezing-mixture is a brine of such a composition as to resist solidification at the temperature of the freezing-mixture. Accordingly we ought to find that the liquid portion of a freezing-mixture has the same percentage composition as the mother-liquor of a saturated brine from which the bihydrate has been separated out by the external application of an ice-salt freezing-mixture. To test how far this is verified by experience, dry ice and salt were mingled in three proportions, namely 3 of ice to 1 of salt, 1 of ice to 1 of salt, and 1 of ice to 3 of salt by weight. The salt and ice were in the finest state of division, and the ice was uniformly and as thoroughly dry as possible. The mixtures were constantly stirred in a wooden bowl for ten minutes, and then thrown upon flannel and pressed through.

Ice.	Salt.		NaCl.
3	1	gave a liquid containing	22·508
1	1	,, ,,	24·343
1	3	,, ,,	24·711

The amount of salt found is therefore not far off that contained in the 10 hydrate, namely 24·5.

Cryohydrates.

§ 18. *General.*—The discovery of the hydrate of chloride of sodium which contains about ten molecules of water to one of the anhydrous salt, caused me to look for similar combinations of water with other salts. This was of course all the more necessary, since if such combinations existed with the salts occurring in the sea, and if such combinations had solidifying-points within the range of the atmospheric temperature, the composition of the solid formed when sea-water freezes would be partly that of ice and partly that of the solid hydrates formed from the brine which had been enriched by the removal of water as ice.

§ 19. It has long been known that the presence of a soluble salt in water depresses the point at which the liquid solidifies (irrespective of the nature of the body separated by solidification). Suppose now we take a solution of the salt *a b* saturated at the ordinary temperature and cool it. We may, above 0° C., either get a separation of the anhydrous *a b*, or some crystalline combination of the salt with water, a hydrate. To this Na Cl is the only exception; for this body is equally soluble in water at all temperatures above 0° C. Putting Na Cl on one side, we may admit, then, that the mother-liquor gets poorer and poorer as the temperature falls. This is obviously the case if the anhydrous salt falls down; and experience informs us that in order to dissolve a hydrated salt, water, and not the anhydrous salt, must be added. Experience also teaches that there is no marked discontinuity at 0° C. We never find the whole of the salt separated before or even at 0° C. One patent consequence of this is that we can form a freezing-mixture (whereof a portion is liquid) by mixing any soluble salt with ice. We get, therefore, in all cases a solution of the salt below 0° C. If as the temperature is still lowered anhydrous salt were to separate out, we should at last get pure water unfrozen below 0° C.—an impossible result. The same would ensue if a hydrate richer in salt than is the solution were to separate out; whereas if ice or a hydrate poorer in salt than is the solution were to separate out, we should then get at a lower and lower temperature a richer and richer solution, and return to the very condition of strength which the lowering of temperature had altered. This is also an impossibility. Accordingly, in all cases some temperature below 0° C. must be reached at which, after separation of the anhydrous salt or a hydrate richer in salt than the solution, the water and salt solidify together, that is, in constant proportion. The effect of such solidification must be (1) the preservation of a constant temperature during solidification from the moment when the proper proportion between the water and the salt is reached; and (2) the preservation of that proportion exhibited by the identity of composition of any crop of the solid and the mother-liquor. It is clear that if ice and the hydrated or anhydrous salt separated out in constant proportion and not combined, but merely mixed with one another, the mass would have a constant solidifying- and melting-point; and this would be below zero to the same amount as would be reached on mixing artificially the anhydrous or hydrated salt with ice in the same proportion. But when we have distinct and unchangeable relation by weight demanded by the constancy of the solidifying- and melting-point, we have undoubtedly a numerical physical relation as fixed, and no less important than the points of fusion or degrees of solubilities. And if, as I shall show, all the hydrates formed under these conditions have distinct crystalline forms, we have all the conditions of chemical association; at least I know of none other. It is an essential element in the existence of these compounds that they can only exist in the solid state below 0° C. Hence I propose to call them for the present "cryohydrates." At the ordinary temperatures they melt in their own water of crystallization, and appear as ordinary solutions never saturated. And when once the proportion between the salt and the water in a cryohydrate has been found, the cryohydrate can be formed in any quantity by dissolving the salt in water in the required proportion. Such a solution shows no sign of yielding up ice or anhydrous salt (or other hydrate) until its temperature, on being lowered, reaches a certain temperature peculiar to the salt (unless under supersaturation); it then solidifies as a whole, maintaining throughout that constant temperature. Above this temperature (that is, in the melted state) it is precisely in the same predicament as a salt melted in its own water of crystallization.

§ 20. The same cryohydrates are (and indeed must inevitably be) formed if we cool such a solution of the respective salt in water as contains a greater proportion of water than the cryohydrate. As we have seen in §§ 10, 13, where the experiments dealt with brine of NaCl, ice is then formed, and the liquid gets richer and richer in salt, falling in temperature till the ratio proper to the cryohydrate is reached; whereupon, as before, homogeneous solidification ensues without abasement of temperature. If we approach the cryohydrate from this side (that is, by removing ice from a dilute solution), we are sure not to run the risk of being inconvenienced by the intervention of any intermediate hydrate similar to the bihydrate of chloride of sodium (§ 11). But such intermediate hydrates are of the rarest possible occurrence; so that on account of the high specific and latent heat of water, it is invariably most convenient to start from a saturated solution.

§ 21. *Cryohydrate of Chloride of Ammonium.*—A saturated solution of NH_4Cl was cooled in ice and the liquid portion transferred to a beaker surrounded by an ice-salt freezing-mixture. The temperature fell continuously, and anhydrous chloride of ammonium kept falling down. At —15° the solidifying part presented a different appearance. It then took the form of a brilliant white apparently flocculent mass lighter than the unsolidified liquid. After standing, with stirring, for a quarter of an hour, the temperature was still −15°. The clear portion was poured off into a fresh beaker, to which the cold was applied. The solidifying parts are now seen to be minute crystals, very much resembling ice-flowers, but opaque. The sides of the beaker become studded with transparent crystals of four sides, which are striated parallel to the sides. By and by these crystals become perfectly white and opaque, and a third axis of crystallization is developed, which was at first suppressed. The crystals are perfectly beautiful, resembling, where opaque, frosted silver. On allowing a thick cup to freeze and breaking it, an exquisite pearly appearance is presented. The structure appears then quite fibrous, the fibres running perpendicular to the axis of the cup; and the appearance, as far as structure is concerned, is similar to that of sublimed chloride of ammonium. The temperature remains constant at −15° C., even to perfect dryness. The first crops of crystals were rejected as being possibly contaminated with NH_4Cl. The last crop and the mother-liquor were analyzed by being weighed into glass basins and evaporated at 100° C.

	grms.		NH_4Cl.		per cent.
I. . .	9·5360	gave	1·7573,	or	18·43
M.L. II. . .	6·0890	„	1·1895	„	19·56

The analysis II., which is of the mother-liquor, corresponds nearly with the molecular relation $NH_4Cl + 12H_2O$, which requires 19·98 per cent. of NH_4Cl.

§ 22. *Cryohydrate of Sulphate of Zinc.*—The hydrates of sulphate of zinc already known are very numerous. They are:—

$$ZnSO_4 + H_2O,$$
$$ZnSO_4 + 2H_2O,$$
$$2ZnSO_4 + 7H_2O,$$
$$ZnSO_4 + 5H_2O,$$
$$ZnSO_4 + 6H_2O,$$
$$ZnSO_4 + 7H_2O.$$

The last, which is the ordinary form of zinc-vitriol, when saturating water at 17°·5 C., gives a solution, according to Karsten, consisting of 52 per cent. of salt and 48 per cent. of water. On cooling such a solution to and below 0° C., the heptahydrate crystallizes out; and this, consisting of 56 of salt to 44 of water, impoverishes the mother-liquor until the latter contains 30·84 per cent. of the anhydrous sulphate, The temperature is now —7° C., and it remains constant at this degree. The last fraction having solidified at this temperature, was remelted and the water estimated by evaporation and heating to 240° C.

8·1531 grms. contained 2·5146 grms. of $ZnSO_4$,
or 30·84 per cent.

This corresponds very closely with the composition

$$Zn\,SO_4+20\,H_2\,O.$$

It is noteworthy that this cryohydrate, after standing some days in a hermetically sealed tube, deposits massive rhombic crystals and a fine powder. I have not analyzed these; they are possibly the monohydrate and one of the intermediate hydrates insoluble in the fused cryohydrate.

§ 23. *Cryohydrate of Sulphate of Magnesium.*—Combinations are known consisting of 1 molecule of sulphate of magnesium combined with 1, 2, 6, 7, and 12 molecules of water. The last is the more interesting because Fritsche describes its formation from a saturated solution of the sulphate when cooled below 0° C. I find that when a saturated solution is cooled to —5° C. and transferred to a clean vessel, it may, if perfectly free from crystals of the 7-hydrate, be cooled to —10° C. without any further solidification. As soon, however, as further cold causes crystallization, the temperature rises to —6° C., and remains constant at this point during the whole of the subsequent solidification, provided that a crystal of the previous crop is put into the cooled mother-liquor after each decantation.

The composition was determined by heating to 240° C.

7·6564 grms. gave 1·6736 grm. of anhydrous sulphate,
or 21·86 per cent.

This corresponds to 23·83 (say 24) molecules of water. Its molecular ratio seems therefore to be

$$Mg\,SO_4+24\,H_2\,O.$$

§ 24. *Cryohydrate of Nitrate of Potassium.*—As far as I can inform myself, nitre, like the chloride of ammonium, has not hitherto been combined with water. The solution saturated at 20° C. gives an abundant crop of nitre at 0° C. There appears to be no intermediate hydrate, the body which separates at −2°·5 C. being apparently pure nitre. At −2°·7 another body is formed, whose crystalline form resembles ice. It adheres to the side of the vessel; when separated, it floats, being lighter than the mother-liquor at −2°·7; but when thrown into water at 20° C., it sinks, showing that it is not ice. The last four crops. of crystals were analyzed. The final one was formed also exactly at −2°·7.

	Liquid.		KNO_3.		per cent.
(1)	8·8030	gave	0·9221	or	10·4
(2)	1·6709	„	0·1959	„	11·7
(3)	3·0300	„	0·3350	„	11·06
(4)	4·2280	„	0·4770	„	11·2

The molecular relation is therefore about 1 : 44·6.

$$KNO_3 + 44H_2O.$$

§ 25. *Cryohydrate of Sulphate of Copper.*—The ordinary hydrates of blue vitriol are the mono-, bi-, and pentahydrates. On cooling a saturated solution, the constant temperature of solidification is found to be −2° C. The last crop of crystals being remelted, was evaporated and heated to the anhydrous state.

6·6952 grms. gave 1·1312 grm. $CuSO_4$, or 16·89 per cent. This corresponds to the atomic relation of 1 : 43·7 (say 1 to 44)

$$Cu\,SO_4 + 44H_2O.$$

Each crop of crystals, when melted, and the mother-liquor present identically the same depth of colour.

§ 26. *Cryohydrate of Sulphate of Sodium.*—A very great many ordinary hydrates of this salt are known. I find that a saturated solution has a solidifying-point at −0°·7 C.

4·0630 grms. contained 1·850 grm. of $Na_2\,SO_4$, or 4·55 per cent.,

corresponding to the molecular ratio of 1 : 165·6 (say 1 to 166),

$$Na_2\,SO_4 + 166H_2O.$$

§ 27. *Cryohydrate of Chlorate of Potassium.*—The chlorate of potassium, like the nitrate, has not hitherto been combined with water. Almost the whole of the salt separates out in the anhydrous state when a saturated solution is cooled to 0° C. On further cooling to −3° C., the solution may present a remarkable condition of double supersaturation. If at this temperature a crystal of anhydrous chlorate is dropped in, anhydrous chlorate is formed in considerable quantity. If an ice fragment is introduced, ice only is formed. If both are thrown in, both are formed, the one set of crystals floating, the other sinking. If the temperature of the supersaturated solution be further cooled, the proper cryohydrate separates out and the temperature rises to −0°·5. The normal formation ensues on introducing a crystal of the cryohydrate from one crop to start the formation of the next. This phenomenon, which is not without its counterpart in some other cases, argues forcibly for the existence of a distinct crystalline form proper to the cryohydrate.

(1) 13·2512 grms. of cryohydrate gave 0·3912 grm. $KClO_3$, or 2·95 per cent.

(2) 13·5810 grms. of cryohydrate gave 0·3915 grm. $KClO_3$, or 2·88 per cent.

The molecular relation is exactly

$$KClO_3 + 222 H_2O.$$

§ 28. *Cryohydrate of Bichromate of Potassium.*—I chose this salt because it is not constructed on the ordinary salt type. I found it solidified entirely at −1° C. The crystals, which are at first transparent, become opaque, and are of a bright straw-yellow.

5·4968 grms. gave 0·2868 $K_2Cr_2O_7$, or 5·305 per cent.

This gives the largest ratio of water, namely

$$K_2Cr_2O_7 + 292 H_2O.$$

As with the sulphate of copper, so in this case all crops of crystals, when melted, had exactly the same depth of colour.

§ 29. *Sources of error.*—I need scarcely point out that the molecular ratios derived from the above experiments can only be considered provisional. In the case of bodies which combine with as many as forty and upwards molecules of water, a very slight error in the determination may make a difference of one or two molecules of water. In those which have a very low solidifying-point, another source of error creeps in. The liquids, even on pouring from one vessel to another, dilute themselves by condensing moisture from the air. With due regard to the probable direction of error in each case, I venture to submit the following tabulation, showing in column 1 the anhydrous salt, in 2 the temperature Centigrade of solidification, in 3 the actual percentage of anhydrous salt found, and in 4 the number of molecules of water to one of salt.

Table VIII.

Temperature of Solidification and Molecular Ratio of Cryohydrates.

(1)	(2) Temp. Cent.	(3) Per cent. of salt.	(4) Molecules of water to 1 of salt.
$NaCl$	−23°	23·603	10·5(? 10)
NH_4Cl	−15	19·56	12·0
$ZnSO_4$	− 7	30·84	20·0
$MgSO_4$	− 6	21·86	24·0
KNO_3	− 2·7	11·20	44·0
$CuSO_4$	− 2	16·89	44·0
Na_2SO_4	− 0·7	4·55	166·0
$KClO_3$	− 0·5	2·93	222·0
($K_2Cr_2O_7$	− 1·0	5·306	292·0)

§ 30. The above are arranged according to the molecular ratio, NaCl having the least number of molecules of water. At once an important law declares itself: those cryohydrates which have the lowest solidifying temperatures have the fewest molecules of water. This law holds true with all which have been examined, with the exception of the bichromate of potassium, which, if placed according to its temperature of solidification, would be above the sulphate of sodium. Is this due to its abnormal composition? Such questions suggest themselves by the score on contemplating this Table VIII.

§ 31. I have found the solidifying-points, and have hermetically sealed specimens, of the cryohydrates of a few other salts, and am collecting more previous to analyzing them. I give their solidifying-points in the order of the degree of cold; and it will be of great interest to see whether their molecular ratios fall in accord with the above rule.

Salt.	Solidifying temperature of cryohydrate.
KI	−23°·0
KBr	−13·0
KCl	−11·4
Na_2CO_3	− 2·0
$BaNO_3$	− 0·8
$H_2C_2O_4$	− 0·5

Perhaps the most promising direction of inquiry for the establishment of such a uniformity of result as may be used for the prediction of untried experiments will consist in a careful study of the nine salts between K, Na, and NH_4 on the one hand, and Cl, Br, and I on the other.

Some Experiments with Sea-water.

§ 32. *Freezing sea-water.*—The sea-water with which the following experiments were performed was procured from Dover. After filtration, it was found to have at 760 millims. the boiling-point 100°·6, while the temperature of its vapour was 100°·2. This sea-water began to freeze at −2° C. On evaporation on a water-bath and keeping at 100° C. for two hours, the percentage of solid residue was 6·5786. A large beaker of this sea-water was cooled to 0° C. A tin vessel was supported inside the beaker so that its bottom just touched the surface of the water; and a freezing-mixture was placed in the tin vessel. When about $\frac{1}{100}$ of the whole had solidified, the solid was removed and divided into two parts: one was allowed to melt, and its percentage of solid matter was determined as above; the other was broken up and frequently pressed between linen and flannel in a screw press, being allowed to melt as little as possible. The percentage of solid matter in this also was determined. The following numbers show the result of this examination:—

	Per cent. at 100° of solid residue.
Sea-water	6·5786
Frozen sea-water	5·4209
Frozen and pressed sea-water	0·4925

It appears, then, that under these conditions the freezing of sea-water is little more than the freezing of ice, and that the almost undiminished saltness of the unpressed ice is due, as suggested by Dr. Rae, to the entanglement amidst the ice-crystals of a brine richer in solid constituents than the original water itself. Such brine, which is here squeezed out in the press, drains in nature down from the upper surface of the ice-floe by gravitation, and also is replaced by osmic action by new sea-water which again yields up fresh ice; so that while new floes are porous and salt, old ones are more compact and much fresher, as the traveller observed.

§ 33. But, bearing in mind the existence of the cryohydrates, certainly of sulphate of magnesium and doubtless also of chloride of calcium at temperatures below 0° C., a rapid fall of temperature may be accompanied by more complex phenomena of gelation; for if the ice be quickly removed from a large mass of water by freezing, the resulting brine may easily be so enriched as to throw out one or more cryohydrates, which thus perpetuate *in situ* a definite amount of saline matter. How far such cryohydrates are soluble in the chief cryohydrate, namely that of NaCl, which by itself resists the cold so long, is an important matter for future research. But there can even, viewed in the light of the experiments given above, be little doubt that the degree of saltness of a floe depends not only upon its age, but also upon the rapidity with which it was at first formed, and upon the lowest temperature to which it has subsequently been exposed.

§ 34. Since sea-water has no maximum density below its freezing-point, when a mass of sea-water is uniformly cooled to −2° C. ice will be formed at any point, whether at the bottom or at the surface, which loses more heat. Even the middle of a mass of sea-water may lose heat by radiation, and crystals of ice be thus formed in the mass. Or the bottom of the sea may radiate sufficient heat through the ice-cold layers above to freeze the water in contact with it, and thus generate large masses of ice which break off and carry with them parts of the sea-bottom. But I suppose the ice of the sea is mainly formed at or near the surface by radiation from the surface into space and by contact with tbe colder air. To imitate as nearly as I could the condition which I suppose to exist, I cooled the sea-water to −2° C. in a beaker, which I enveloped thickly with flannel. I tried in vain to freeze the surface by blowing over it dry air which had passed immediately before through a long pewter worm immersed in a freezing-mixture. But I succeeded in getting a sheet of ice when I hung a freezing-mixture contained in a blackened tin pan within about $\frac{1}{8}$ inch of the surface of the water, the whole being plentifully enveloped in flannel. Perhaps here the actual conditions which obtain when sea-water freezes were reproduced. I found that the pressed ice contained only 0·4052 of solid residue at 100° C.

§ 35. The question suggested itself to me whether, when one part of a solution of a salt is cooled, there may not be an accumulation of salt in the cooler part, although not accompanied by any solid separation. I accordingly cooled a saturated solution of nitre to $-1°$ C. and decanted from the separated nitre. I then warmed the solution in a tall beaker-glass to 60° C. and placed the bottom of the beaker in melting ice. In an hour's time a thermometer at the bottom stood at 10° C., at the top at 33°. A specimen from the bottom contained 11·3 per cent. of nitre; one from the top contained 11·7 per cent. of nitre—showing that there was no sensible diffusion of the salt one way or the other.

General Considerations.

§ 36. *Maximum density of Water.*—It was shown that brines of various strength, when mixed with water, absorbed heat and expanded. Let us look upon ice as the cryohydrate of water. Water shrinks as it loses heat till it reaches 4° C. At this point ice is formed, which, however, is dissolved in the water. A solution is obtained having a temperature of solidification below 4° C., namely at 0° C. At 0° C. the ice and the water solidify together, producing the compound body or cryohydrate called ice, which is thus a cryohydrate of water. The expansion from 4° to 0° is due to the greater and greater amount of ice which the water holds in solution, and which expansion is greater than the contraction of the water due to the diminished temperature.

§ 37. *Variation of media.*—There can be no doubt that the discovery of an enormous number of new bodies of definite composition will reward those who labour in this field. Taking water as the medium for solution, there appears to be no doubt that every soluble salt has a definite cryohydrate; so that in this direction alone the number of new bodies awaiting discovery and description may be estimated at half the number of bodies already known. If we vary the medium, employing, say, alcohols or hydrocarbons as solvents, the number of new compounds will be again indefinitely increased; so that it is fairly within the truth to assert that the number of known bodies may soon be doubled.

§ 38. *Geological.*—The behaviour of mixtures of salts will again offer a new chapter for study; and I suppose we may expect that much light will be thus thrown upon some of the most obscure geological questions. For though we have been considering above cryohydrates (that is, compounds of water solidifying below the freezing-point of water), there can be no discontinuity separating the medium water with its peculiar temperature of solidification from other media of very different melting-points. We know already, indeed, very many instances in which the mixture of two bodies has a lower melting-point than either of its constituents. What must happen, then, if a mass of molten rock, such as a silicate, is saturated at a high temperature with another silicate? When the mixture cools, the second may separate out in the solid form, perhaps as quartz, perhaps as felspar, or what not. Anon, at a certain lower temperature, solidification takes place between the medium and the dissolved rock in definite proportion—definite, though perhaps not necessarily in chemical ratio, but presenting that mineralogical ratio which is so striking, and which has not hitherto been satisfactorily explained.

§ 39. *Constant temperatures.*—Perhaps one of the most interesting aspects of the experimental results is the establishment of fixed temperatures below zero. With the exception of the melting-points of a few organic bodies such as benzol, and the boiling-points of a few liquids such as liquid ammonia, sulphurous acid, and carbonic acid, and the rather ill-defined temperatures to be got by various freezing-mixtures, there are no means in the hands of physicists for obtaining and maintaining with certainty and ease a fixed temperature below 0° C. Now, if we surround a body with one of the solid cryohydrates described above, the body is kept at a corresponding temperature as long as any of the cryohydrate remains solid, and this with as much certainty as the temperature 0° C. can be maintained by melting ice. We thus command temperatures between −23° and 0° C. with the greatest precision.

§ 40. *Invitation to others.*—I need scarcely point out that the field of inquiry which has been here opened is far too large to be satisfactorily examined by one worker. It is notably at the commencement that the collaboration of many workers is most beneficial, so that fundamental errors may be quickly corrected. On this ground I respectfully invite my fellow-labourers to this branch of inquiry.

I have received through a considerable part of this inquiry much valuable assistance from my friend Mr. F. H. Marshall.

January 18, 1875.

XII. *On Salt Solutions and Attached Water.*

By FREDERICK GUTHRIE.

II.

Cryogens and Cryohydrates.

BY Cryogen I mean an appliance for obtaining a temperature below 0° C. In this paper it always signifies a freezing-mixture. By Cryohydrate I mean the body resulting from the union of water with another body, and which hydrate can only exist in the solid form below 0° C. As this communication is in every respect continuous with the one brought before the Physical Society on November 7th, 1874, and published in the Philosophical Magazine for January 1875, the paragraphs are numbered in sequence with those of the previous communication.

CRYOGENS.

Precision of Temperature of Freezing-mixtures.

§ 41. In § 39, when I was speaking of the possible uses of cryohydrates for the maintenance of constant temperatures, I said:—"With the exception of the melting-points of a few organic bodies such as benzol, and the boiling-points of a few liquids such as liquid ammonia, sulphurous acid, and carbonic acid, and the rather ill-defined temperatures to be got by various freezing-mixtures, there are no means in the hands of physicists for obtaining and maintaining with certainty and ease a fixed temperature below 0° C."

In regard to freezing-mixtures, I confess to have been here very much misled by the confident but rather erroneous statements of others, to which I attached faith trebly blind—blind, because no recorded experiments really support them, blinder still because a little thought in the right direction must have shown their fallacy, and blindest of all because the one experiment of my own in this direction (§§ 16 and 17) shows that the minimum temperature of an ice-salt cryogen is reached whether we take the ratios 3 of salt to 1 of ice or 1 of salt to 2 of ice, and so points to the wideness of the margin of ratio which may obtain between the weights of ice and the salt. It will further be shown in §§ 44–46 that the temperature of the constituents has nothing to do with the temperature reached, and in § 49 that the degree of hydration of the salt employed is often without effect.

Accordingly I unqualifiedly withdraw the expression "ill-defined" as applied to freezing-mixtures. They may be, on the contrary, bodies of precise temperature under widely varying circumstances.

§ 42. The enormous latent heat of water, the fact that the specific heat of ice is only about half that of water, while the specific heats of all salts are far less than that of ice, and therefore, *à fortiori*, less than that of water, together with the good thermal conductivity of water, all argue that, if constantly stirred, all parts of a freezing-mixture will have the same temperature. The fact that the liquid portion of a freezing-mixture of ice and a solid salt is the cryohydrate of that salt, ensures the identity of the resulting temperature under various conditions of proportion. The constant tendency to the formation of this cryohydrate by contact between the solids is always seeking to depress the temperature; while the solidification of the cryohydrate at an indefinitely small fraction of a degree below the temperature of the freezing-mixture, and the consequent liberation of heat, ensures the temperature against such fall.

Statements therefore, whether previously made by myself or others, that it is advantageous to weigh the salt and ice in definite proportion, that the ice should be dry, that snow is preferable to ice on account of its state of finer division, that additional cold is produced by previously cooling the ice or salt or both, are to be put aside as untrue—untrue, that is, as far as the temperature or heat-tension is concerned. To obtain the greatest quantity of heat-absorption with a given amount of salt, such a quantity of ice must be taken as will form with the salt a cryohydrate. The proportions can be at once gathered from Table X. § 90.

§ 43. Further, the fact that all cryohydrates, with, I believe at present, the sole exception of sulphate of zinc and chloride of magnesium, have far more water than that ordinary hydrate which has most water, shows that it cannot matter whether a salt which affects water of crystallization be employed in the anhydrous state or with its crystalline water.

It may, however, be otherwise with salts containing the elements of water more intimately associated; and, as we shall see in one case at least, an anhydrous compound may melt ice with which it is in contact and heat the so-formed water far above zero, while the compound so formed, when cool, will when mixed with a fresh quantity of ice absorb heat abundantly. I suppose in such cases double decomposition ensues; and though nothing is eliminated, there is a rearrangement of the elements more intestinal than that effected by the association with water.

§ 44. *Effect of the Temperature of the Ingredients on that of the Freezing-mixture.*—An ounce of finely powdered chloride of sodium was cooled in a flask surrounded by a freezing-mixture till its temperature was −15° C. It was then stirred with four ounces of ice, which had been cooled and had the temperature −10°. As soon as liquefaction began, the temperature −22° was reached; and this degree of cold was never surpassed.

§ 45. The same degree of cold (−22°) resulted from the mixture of 1 oz. of $NaCl$ at −15° with 4 oz. of ice at 0°, also when 1 oz. of salt at +12° C. was mixed with 4 oz. of ice at −12° C.

§ 46. Indeed the margin of temperature may be greatly extended. Thus, 1 oz. of $NaCl$ in powder was heated to incipient redness and thrown upon 5 or 6 oz. of ice at 0°; after a few minutes constant stirring, the temperature had reached −22°.

§ 47. 1 oz. of dry anhydrous $Na_2 SO_4$ was heated nearly to redness, and thrown upon 4 oz. of ice at 0°. In a few minutes the temperature had sunk to −0°·7. Again, an ounce of anhydrous $CuSO_4$ was heated to about 600° C., and thrown upon

4 oz. of ice; the temperature at once sank to $-0^{\circ}\cdot5$ (compare § 55).

§ 48. From the above experiments, and from the theoretical considerations touched upon in § 42, I conclude that, within very wide limits as to quantity, the temperature of a freezing-mixture may be very independent of the temperature both of the salt and of the ice.

§ 49. *Effect of Crystalline Water in the salt on the Temperature of the Freezing-mixture*—From § 26 it appears that crystallized sulphate of soda ($Na_2SO_4+7H_2O$) gives with ice a temperature of $-0^{\circ}\cdot7$. Deprived of water, the anhydrous salt N_2SO_4 gave with ice the temperature also of $-0^{\circ}\cdot7$. Sulphate of copper in the anhydrous state produced, when mixed with ice, a temperature of $-1^{\circ}\cdot7$, while with the ordinary crystallized hydrate the temperature was -2°.

§ 50. Having examined six or seven other salts as to the temperature of their freezing-mixtures when employed both with and without their crystalline water, I do not scruple to assert that water of crystallization, properly so called, takes no effect upon the temperature of the freezing-mixture; and I believe the same is true of that water which has been called constitutional. But where a profound rearrangement of the elements of water ensues, the effect may be different, as appears in the next paragraph.

§ 51. The chloride of aluminium, $AlCl_3$, as offered in commerce, fumes strongly in the moist air, so great is its avidity for water. Thrown upon several times its own weight of ice, the two liquefy and the temperature stands above 0°. The strongest commercial solution of chloride of aluminium, however, when at the temperature of the air or at 0° or at 100° C., will reduce the temperature to -13° when poured upon three or four times its own weight of ice. I suppose the anhydrous chloride may be viewed as separating the atoms of the water-molecule, as is supposed to be the case with the chlorides of phosphorus.

§ 52. *Hydrochloric Acid as a Cryogen with Ice.*—Ordinary commercial hydrochloric acid when poured upon ice may produce as much as 22° of cold. The amount of HCl in the hydrochloric acid, however, is here to be as much considered as it would have to be if we dealt with ice and a solution of NaCl. In the experiments which are given in the succeeding paragraphs as far as § 54, the hydrochloric acid employed was formed by saturating water with the gas, while the liquid was kept after absorption had ceased for half an hour at 0° while the acid passed through. The barometric pressure was 770 millims. This may be considered a normal acid.

§ 53. The acid at 0° was poured in various proportions upon ice at 0°. Fifty grms. of ice were used in each experiment. The Table shows within what small limits of ratio the minimum temperature is reached. In the Table the weight of the ice is taken as unity.

Weight of ice at 0° C.	Weight of $HCl+nH_2O$ at 0° C.	Resulting Temperature.
1	1·5	− 3°
1	0·4	−26
1	0·3	−23
1	0·2	−19

We are therefore, when dealing with a cryogen one of whose constituents is a liquid, much more limited in the range of ratio which we may employ to procure the maximum cold than is the case when both are solid.

§ 54. Accordingly, in examining the effect of the alteration in the temperature of one or both of the constituents, it is necessary always to use the same ratio. In the Table below, the ratio used was that which gave the maximum cold when both were at 0° C., namely 1 of ice to 0·4 of the saturated solution of HCl. It must be understood that in those cases where the hydrochloric solution was used below 0° C., it is not meant that the saturation with HCl took place at that lower temperature, but that, having been saturated at 0°, the solution was subsequently cooled to the lower temperature.

1 ice at −15° with 0·4 HCl sol. at 0° gave −28°.
1 „ 0° „ 0·4 „ „ −15° „ −27°.
1 „ −15° „ 0·4 „ „ −12° „ −27°.

All these numbers are sufficiently near to −26°, the number got when both ingredients are at 0°, to justify the conclusion that the temperature of the ice has something, but little, to do with the temperature of the freezing-mixture. And it appears that the temperature of the HCl solution is also without effect, provided that the saturation has been effected at the same temperature and pressure. The water acts, indeed, here much as the water of crystallization acts in the case of solid salts.

§ 55. *Table of Freezing-mixtures.*—In the following Table the lowest attainable temperature is given. This, as we have seen, is independent of the temperature of the salt and its degree of crystalhydration. The temperatures are got on mixing the salt with three to six times its weight of ice in lumps of the size of a pea downwards. The salts are arranged according to the degree of cold they furnished.

TABLE IX.—Freezing-mixtures.

	° C.		° C.
NaBr	−28·0	KCl	−10·5
NH_4I	27·0	K_2CrO_4 . . .	10·2
	28·0	$BaCl_2+2H_2O$.	7·2
NaI	26·5	$Sr2NO_3$ . . .	6·0
$CuCl_2$	24·5	$MgSO_4+7H_2O$.	5·3
KI	22·0	$ZnSO_4+7H_2O$.	5·0
NaCl	22·0	KNO_3	3·0
$MgCl_2+6H_2O$.	20·5	Na_2CO_3 . . .	2·2
$SrCl_2+6H_2O$.	18·0	$CuSO_4+5H_2O$.	2·0
$2NH_4, SO_4$. . .	17·5	$FeSO_4+7H_2O$.	1·7
NH_4Br	17·0	K_2SO_4	1·5
NH_4NO_3 . . .	17·0	$K_2Cr_2O_7$ . . .	1·0
$NaNO_3$	16·0	$Ba2NO_3$ . . .	0·9
NH_4Cl	16·0	$Na_2SO_4+10H_2O$	0·7
$FeCl_3$ (commercial)	16·0	$KClO_3$	0·7
$Ca2NO_3+4H_2O$.	14·0	$AlNH_4 2SO_4+12H_2O$	0·4
KBr	13·0	$HgCl_2$	0·2
$AlCl_3$ in strong sol.	13·0	NH_4CO_2 . . .	0·2

We shall study this Table in connexion with the temperature and molecular ratios of the cryohydrates of the same salt.

CRYOHYDRATES.

Cryohydrates of the Halogen Alkalies.

§ 56. As was anticipated in § 31, the study of the nine salts resulting from the union of the halogens with the alkali metals has brought to light many points of interest. The two members of this family which I had previously examined were NaCl, solidifying at −23° with 10·5 molecules of water, and NH_4Cl, solidifying at −16° with 12 molecules of water. I have reexamined the cryohydrate of chloride of ammonium; and the mean of several analyses gives 19·27 per cent of chloride, which brings the ratio to $NH_4Cl + 12\cdot4 H_2O$.

There is nothing particular to remark in the analysis of this group. All, excepting the ammonium salts, bear heating to 300° or 400° C. without decomposition or volatilization. The liquid cryohydrates were weighed in covered basins of Bohemian glass, evaporated to dryness, strongly heated, covered, and quickly cooled. The ammonium salts were heated on the water-bath till they ceased to lose weight.

§ 57. *Iodide of Potassium.*—This salt, which at the ordinary temperature is so abundantly soluble in water, yields a large proportion of anhydrous salt when cooled. An ice-salt cryogen fails to solidify it, but just brings it to the verge of solidification, removing all the free salt in the same manner as it does with NaCl solution, excepting that there does not appear to be any intermediate cryohydrate corresponding with $NaCl + 2H_2O$. The KI solution appears to be homogeneous at $-8°$ or $-12°$. When subjected to solid CO_2 and ether, it at once begins to solidify at $-22°$ to $-23°$, and retains this temperature to dryness. Of the four parts into which the substance was divided as it solidified, two were analyzed—namely, the second crop of crystals, and the finally solidified and remelted mother-liquor. Of the first, 5·8240 grms. gave 3·0120 anhydrous KI, or 51·72 per cent.; of the second, 6·3960 grms. gave 3·331, or 52·07 per cent. The latter corresponds to the molecular ratio

$$KI + 8\cdot5 H_2O.$$

§ 58. *Bromide of Potassium.*—This forms a cryohydrate of great beauty. At $-13°$ the characteristic crystals are seen to be produced. They have the fern-like shape of ice-crystals. The fronds are studded with a fructification of opaque crystals, whose opacity seems to spread through the at first transparent fronds till the whole resembles frosted silver. The two portions which were taken for analysis were the final part, or that which solidified last, and the immediately preceding crop of crystals. Of the former, 7·8285 grms. gave 2·5170 of KBr; of the latter, 9·4560 grms. gave 3·0070 dry salt. These correspond respectively with 32·15 and 31·80 per cent. The first points to the molecular relationship

$$KBr + 13\cdot94 H_2O.$$

§ 59. *Chloride of Potassium.*—Since KI is thus seen to combine with 8·5 of water and KBr with 14 of water, the one a fraction and the other a whole number, I spared no care in the preparation and analysis of the chloride of potassium, to see whether the fractional relationship ~~did~~ not arise from experimental error. Perfectly pure K_2CO_3 was dissolved in a slight excess of HCl, evaporated to dryness, and ignited in a platinum basin. The KCl was recrystallized four times. Six ounces of the saturated solution were cooled; solid matter separated down to −10°. At this temperature the separated salt appeared as a cloudiness in the liquid, a little heavier than the latter, and of a granular but translucent appearance. The liquid may now be cooled to −15°; but it is now a supersaturated solution of the true cryohydrate—a solution, it is to be observed, of which the solid crystals present are unable to determine the solidification. At a little below −15°, long ice-like crystals shoot out and the temperature rises to −11°·4, which is the crystallizing-point of the cryohydrate. From this behaviour I am disposed to admit that there may be an intermediate hydrate similar to the bihydrate of NaCl.

Of the last portion which solidified, 6·588 grms. gave 1·320 of anhydrous salt. Of the previous crop of crystals 7·869 grms. gave 1·579 of salt. These show the respective percentages 20·03 and 20·07, both of which point to the relationship

$$KCl + 16{\cdot}5\ H_2O.$$

§ 60. The halogen salts of potassium arrange themselves according to the cold required for the solidification of their cryohydrates, and according to the molecular ratio of water, as follows:—

	Solidifying-temperature of cryohydrate.	Number of molecules of water to 1 of salt.
KI . . .	−23°	8·5
KBr . . .	−13	14·0
KCl . . .	−11·4	16·5

We find here the general rule confirmed, that among like salts the lower the temperature of solidification of the cryohydrate, the smaller is the number of molecules of water which it contains.

§ 61. *Iodide of Ammonium.*—This salt gave me considerable trouble. It is difficult to obtain in perfectly colourless crystals; and both the solid and its solution are disposed to change colour, becoming brown when kept, even in the dark. Heating in a water-bath with constant stirring restores the dry altered salt to a light-grey colour. Analysis showed that this coloration did not sensibly affect the percentage composition of the salt; but still its power of assuming water might be seriously affected. A saturated solution was exposed for some hours to an ice-salt cryogen at $-22°$; it was then exposed to the carbonic-acid-and-ether cyanogen. It solidified wholly at from $-27°$ to $-28°$. While solidification is taking place, the cryohydrate is nearly perfectly white. When dry and over-cooled, it assumed a pinkish grey colour, resembling chloride of silver which is beginning to be affected by light.

The analysis of these portions is given, namely:—(1) the last to solidify or remelted mother-liquor; (2) the immediately preceding crop of crystals; and (3) the crop before (2). As the results are by no means in good accord, I give them all:—

	Solution.		NH_4I.		per cent.
(1)	5·2935	gave	2·9375,	or	55·49
(2)	5·4960	„	3·2120,	„	58·42
(3)	8·4560	„	4·8675,	„	57·56

The portion (1), which, being the last, is in one sense likely to be the most homogeneous, corresponds to the relationship

$$NH_4I + 6·44H_2O.$$

The portions (2) and (3) would indicate as much as 0·7 molecule less of water.

§ 62. *Bromide of Ammonium.*—This salt separates as a cryohydrate from a saturated solution at $-17°$ C. Its formation is well marked, and its analysis was satisfactory. Of the final portion, 4·2591 grms. gave 1·3680 of NH_4Br. Of the immediately preceding crop of crystals, 5·5990 grms. gave 1·8010 of NH_4Br. The percentages are 32·12 and 32·17 respectively: these numbers correspond to the relationship

$$NH_4Br + 11·15H_2O.$$

§ 63. Taking the value of the cryohydrate of chloride of ammonium from § 56 as $NH_4Cl + 12·4H_2O$, we have for the halogen salts of ammonium:—

	Solidifying-temperature of cryohydrate.	Number of molecules of water to 1 of salt.
NH_4I	−27°·5	6·4
NH_4Br	−17	11·1
NH_4Cl	−16	12·4

whence it is gathered that the relative faculties of chlorine, bromine, and iodine, in depressing the temperature at which the cryohydrates are formed, are similar with ammonium to their faculties with potassium; namely, with both, iodine gives a lower-solidifying cryohydrate than bromine, and bromine lower than chlorine; and we have here also confirmation of the rule that in the same series the aquavalents diminish with the temperature of solidification.

§ 64. *Bromide of Sodium.*—To complete this series I take now the halides of sodium. The bromide of sodium separates from a saturated solution as a cryohydrate at −24° C. Taking the last and the next to the last portions, 7·5010 grms. of the former gave 3·1000 of anhydrous bromide; of the latter, 8·3160 grms. gave 3·4605 of NaBr. The corresponding percentages are 41·33 and 41·61, the first of which indicates 8·12 and the second 8·08 molecules of water to 1 of the salt.

$$NaBr + 8{\cdot}12\,H_2O.$$

§ 65. *Iodide of Sodium.*—A saturated solution of iodide of sodium presents in the most remarkable manner the phenomenon of supersaturation. It may be cooled to −22°, freely exposed to the air, and shaken without solidifying. On being placed in contact with solid carbonic acid and ether, it solidifies, and its temperature instantly rises to −15°. The solidification once started by this extreme cold, the temperature remains constant at −15°. Fragments of the cryohydrate so formed induce solidification in other portions subjected to the ice-salt cryogen. The last two portions were analyzed. Of the very last, 6·4000 grms. gave 3·8050 of NaI, or 59·45 per cent. Of the next to the last, 10·2450 grms. gave 6·0845, or 59·39 per cent. The molecular ratio of the cryohydrate is accordingly

$$NaI + 5{\cdot}82\,H_2O.$$

Correlating the iodide and bromide of sodium with the chloride, we have, therefore,

	Solidifying-temperature of cryohydrate.	Number of molecules of water to 1 of salt.
NaI . .	−15°	5·82
NaBr . .	−24	8·12
NaCl . .	−23	10·5

§ 66. It appears that the iodide of sodium forms the first exception we have yet met with to the general rule; for while its solidifying-point is above that of its companions, it attaches to itself a less number of water-molecules. In the Table below the whole nine combinations are shown; and they are there arranged according to the number of molecules of attached water, or in what might be called their "aquavalents" if this expression were not too symphonous with "equivalents." Say, therefore, "water-worths."

Salt.	Temperature of solidification of cryohydrate.	Water-worth.
NaI	−15°	5·8
NH_4I	−27	6·4
NaBr	−24	8·2
KI	−23	8·5
NaCl	−23	10·5
NH_4Br . . .	−17	11·1
NH_4Cl . . .	−16	12·4
KBr	−13	14·0
KCl	−11·4	16·5

It appears that, for the same halogen, a sodium salt attaches less water than an ammonium salt, and the ammonium salt less than the potassium salt. Also, for the same metal, an iodine salt attaches less water than the bromine salt, and the latter less than the chlorine salt.

Or if we denote by ^{w}XM the number of molecules of water attached to one molecule of XM, then

$$^{w}X\,Na < {}^{w}X\,NH_4 < {}^{w}XK,$$

and

$$^{w}IM < {}^{w}BrM < {}^{w}ClM.$$

This remarkable rule has no exception amongst the above nine salts; and the only exception to the general rule, that the lower the solidifying-point the fewer the molecules of associated water, is offered by the iodide of sodium. Concerning this see § 68.

§ 67. It may be perhaps more than accidental that the numbers of molecular water-worths show a distinct tendency to be multiples of 0·5. For my own part, recognizing the possible range of analytical error, I for the present distinctly forbear to express any opinion as to whether we are here dealing with the same physical force which constitutes chemical attraction, and which regulates the integral ratios of molecular combination as most chemists appear to understand the term—or whether it is a distinct or distinctly conditioned force binding the salt and water together in quite a new ratio, or a ratio which can only be brought to the chemical one by multiplication by constants, at present arbitrary.

It is useful, however, to reflect that almost innumerable instances are known in which the molecular ratio between a salt and its ordinary water of crystallization is not a simple one. On examining the published determinations of water of crystallization effected by the chemists who are or have been both accurate and scrupulous, we find that in many cases a less simple numerical ratio between the salt and its water would often correspond with the derived result far better than the ratio which has been thence deduced.

§ 68. With regard to the solidifying-point of the cryohydrate of iodide of sodium, I may here at once mention the exceedingly interesting fact which will be discussed in its proper place, that while the cold of a cryogen formed by mixing with ice any one of the other of these eight salts is so closely near as to be considered identical with the solidifying point of the corresponding cryohydrate, the cold of a freezing-mixture consisting of ice and iodide of sodium far exceeds −15°, and in fact reaches −28°, nearly the lowest temperature which I have yet got. On this ground it would be really entitled to be placed at the head of the list of the nine, where its water-worth has placed it in Table (§ 66).

§ 69. *Speculation concerning* NaI.—It is clear that when a freezing-mixture is in action, the liquid portion is a solution of the salt of such a strength that it resists solidification at the temperature of the mixture. Accordingly there must be a cryohydrate of sodium which remains liquid down to −28°. Is its crystalline form so peculiar as not to respond by solidification to any solid particles in the air, and so to be an exceptionally persistent instance of supersaturation? Perhaps the cryohydrate can only exist in the liquid form. When it loses heat, it does not solidify as a whole, but in two parts, each of which is less soluble than the two together. For this reason the mixed solid separated would in all its stages have the same composition as the simple cryohydrate, while its separation, when once begun, might keep itself in activity. The heat-tension then exhibited would be the mean of the temperatures due to the solidification of each constituent.

I can scarcely assert that this is a satisfactory or even to my own mind a clear explanation of the phenomenon. The difficulty would be to some extent removed if we could get evidence of an intermediate cryohydrate resembling that which exists with NaCl. Of such a cryohydrate I have no substantial evidence to offer at present.

Cryohydrates of Alkaline Sulphates.

To see whether the noticeable relation between sodium, ammonium, and potassium is valid in other compounds of these metals, their sulphates were examined.

§ 70. *Sulphate of Ammonium.*—This body, when dissolved to saturation in water, yields abundantly the ordinary hydrated sulphate of ammonium when cooled to 0° C. The separation stops shortly below 0° C.; and the solution may be cooled to −22° while the liquid remains quite clear. This possible depression of the temperature in presence of one hydrate is clearly a happy incident in the genesis of the cryohydrate as far as the assurance of the freedom of the latter from the former is concerned. On cooling by means of carbonic acid and ether, solidification ensues shortly below −22°, and the temperature rises to −17°, at which it remains constant. Of the portion last to solidify, 7·0105 grms. on evaporation to dryness and heating to 130°, gave 2·9280 of $2NH_4, SO_4$, or 41·7 per cent. Of the immediately preceding portion, 4·0695 grms. gave 1·7195, or 42·6 per cent. The first of these corresponds to the relation

$$2NH_4, SO_4 + 10{\cdot}22\,H_2O.$$

§ 71. *Sulphate of Potassium.*—The saturated solution of this salt solidifies at −1°·2. Of the last portion, 3·9095 grms.

yielded 0·3075, or 7·8 per cent. of K_2SO_4. Of the immediately preceding part, 5·2905 grms. gave 0·3960, or 7·5 per cent. The first of these shows the molecular relation

$$K_2SO_4 + 114{\cdot}2\,H_2O.$$

§ 72. *Sulphate of Sodium.*—This was examined in § 26. Its cryohydrate is formed at $-0^{\circ}{\cdot}7$. The percentage of Na_2SO_4 in the last portion was found to be 4·55, and the water-worth 165·6.

§ 73. Accordingly we have the sulphates of the alkalies arranged as follows:—

	Temperature of solidification of cryohydrate.	Water-worth.
$2NH_4, SO_4$ . .	$-\overset{\circ}{17}$	10·2
K_2SO_4 . . .	$-1{\cdot}2$	114·2
Na_2SO_4 . . .	$-0{\cdot}7$	165·6

Once more, therefore, the lower the temperature of solidification of the cryohydrate, the less is its water-worth. But the order is different with SO_4 from the order with a halogen.

Nitrates of the Alkalies.

§ 74. *Nitrate of Ammonium.*—The nitrate of ammonium separates as a cryohydrate from a saturated solution at $-17^{\circ}{\cdot}2$ in exceedingly beautiful fern-like crystals. Dried at 100° C. and heated to incipient fusion, 7·6100 grms. of the part last to solidify gave 3·3265 of the anhydrous salt. Of the next to the last, 7·2755 grms. gave 3·1475. The former therefore contained 43·71 per cent. of NH_4NO_3, the latter 43·26. The first indicates the relation

$$NH_4NO_3 + 5{\cdot}72\,H_2O.$$

§ 75. *Nitrate of Sodium.*—The temperature at which this cryohydrate is formed was found by experiment to be $-17^{\circ}{\cdot}5$; but, for reasons partly considered in § 69, I believe this temperature to be too low, and that, when supersaturation does not intervene, the temperature is $-16^{\circ}{\cdot}5$. Here, however, I assign to it the temperature got by direct observation. Of the last portion to solidify, 5·4210 grms. gave 2·2140 of $NaNO_3$, or 40·8 per cent., which indicates the composition

$$NaNO_3 + 8{\cdot}13\,H_2O.$$

Of the previous crop of crystals, 6·9820 grms. gave 2·8850 of $NaNO_3$.

§ 76. *Nitrate of Potassium.*—This was examined in § 24. The cryohydrate solidifies at −2°·6, and contains 11·2 per cent. of nitre. The water-worth is expressed by the relation

$$KNO_3 + 44{\cdot}6 H_2O.$$

§ 77. The nitrates of the alakalies therefore arrange themselves as follows:—

	Temperature of solidification of cryohydrate.	Water-worth.
NH_4NO_3 . .	−17°·2	5·72
$NaNO_3$ . . .	−17·5 (16·5)	8·13
KNO_3 . . .	− 2·6	44·6

I believe that here again the water-worth really falls with the temperature of solidification. But the order of the salts in regard to their water-worths is again different from the order observed with the sulphate, as well as from that of the chlorides of the same metals.

Chlorides of the Alkaline Earths.

§ 78. *Chloride of Barium.*—The chloride of barium in saturated solution becomes a solid cryohydrate at −8° C. Of the portion last to solidify, 6·6790 grms. gave, on evaporation and heating to 350° C., 1·5490 $BaCl_2$. This corresponds to 23·2 per cent. Of a previous crop of crystals, 6·1350 grms. gave 1·4735 $BaCl_2$, or 24·0 per cent. The first of these shows the formula

$$BaCl_2 + 37{\cdot}8 H_2O.$$

§ 79. *Chloride of Strontium.*—This salt forms a cryohydrate at −17°. Of the final portion, 5·8325 grms. contained 1·6085 of $SrCl_2$, or 27·57 per cent. The crop before this showed 1·3655 grm. of salt in 4·9675 solution, or 27·5 of $SrCl_2$. We may therefore deduce the formula

$$SrCl_2 + 22{\cdot}9 H_2O.$$

Nitrates of the Alkaline Earths.

§ 80. *Nitrate of Barium.*—The cryohydrate of this salt solidifies from a saturated solution at −0°·8. I have only one determination of its composition; namely, 3·0370 grms. gave 0·1610 of anhydrous salt, which corresponds to 5·30 per cent., and the molecular ratio

$$Ba\,2\,NO_3 + 259{\cdot}0\,H_2O.$$

§ 81. *Nitrate of Strontium.*—On cooling a saturated solution to −6° the cryohydrate solidifies. Of the last portion, 6·8010 grms. gave 1·7675, or 25·99 per cent. of $Sr2NO_3$. Of the previous crop of the cryohydrate, 4·6715 grms. contained 1·2105 of the anhydrous salt, or 25·91 per cent. These determinations concur towards the relationship

$$Sr\,2\,NO_3 - 33{\cdot}5\,H_2O.$$

Miscellaneous Cryohydrates.

§ 82. *Sulphate of Iron.*—Green vitriol separates from a saturated solution at −2°·2. The next to the last portion of the cryohydrate showed the following composition:—4·7000 grms. gave 0·8155 $FeSO_4$, or 17·35 per cent. Of the last portion, 4·8140 grms. gave 0·8155, corresponding to 16·94 per cent., and showing the ratio

$$Fe\,SO_4 + 41{\cdot}41\,H_2O.$$

Although the solution of this salt was quite clear when hermetically sealed for analysis in a tube containing not more than ·5 cubic centim. of air, a considerable deposit of sesquioxide took place. The analysis, therefore, is probably not quite exact.

§ 83. *Chromate of Potassium.*—The cryohydrate solidifies at −11°. Of the last crop of crystals, 3·0690 grms. gave 1·1145 anhydrous salt, or 36·27 per cent. Of a previous crop, 5·4775 grms. gave 1·9945, or 36·41 per cent. The first indicates the formula

$$K_2\,CrO_4 + 18{\cdot}8\,H_2O.$$

From § 28 the bichromate has a cryohydrate solid at −1° and containing 292 molecules of water. Accordingly

	Temperature of solidification.	Water-worth.
Chromate, $K_2\,Cr\,O_4$.	−11°	18·8
Bichromate, $K_2\,Cr_2\,O_7$.	− 1	292·0

§ 84. *Ammonium Alum.*—This body was examined on account of the large percentage of water which is held by the ordinary hydrated salt. The cryohydrate solidifies at −0°·2. The part last to solidify was examined; of this, 4·3845 grms. contained 0·2060 grm. of anhydrous alum. This shows 4·7 per cent. Of the immediately preceding crop of crystals, 5·2400 grms. gave 0·2220 of anhydrous alum, or 4·2 per cent. The first of these determinations indicates the relationship

$$Al_2NH_42SO_4+261{\cdot}4H_2O.$$

§ 85. *Perchloride of Mercury.*—A saturated solution of corrosive sublimate solidifies at −0°·2. The ultimate and penultimate portions were examined. Of the former, 6·0640 grms. gave 0·197, or 3·24 per cent.; of the latter, 4·977 grms. gave 0·164, or 3·29 per cent. The first indicates the formula

$$HgCl_2+450\ H_2O.$$

§ 86. *Oxalate of Ammonium.*—This solidifies as a cryohydrate at −0°·2. Of the last portion to solidify, 4·340 grms. contained 0·125, or 2·8 per cent. of the anhydrous salt. This agrees with the relation

$$NH_4CO_2+239{\cdot}1H_2O.$$

§ 87. *Carbonate of Sodium.*—After igniting the carbonate so as to decompose any bicarbonate, the saturated salt solidifies as a cryohydrate at −2°. Of the final portion, 6·4090 grms. contained 0·383 of Na_2CO_3. This shows 5·97 per cent., or the relation

$$Na_2CO_3+92{\cdot}75\ H_2O.$$

§ 88. The following Table shows at a glance the relation between the lowest attainable temperature when the salt is mixed with ice, the temperature of the solidification of the cryohydrate, the water-worth or aquavalent. Column 1 shows the salt employed, and the degree of hydration when associated with water of crystallization. Column 2 shows the temperature obtained when the salt is mixed with ice. Column 3 shows the temperature at which the cryohydrate separates. In column 4 are shown the number of molecules of water associated with one molecule of the salt in the cryohydrate. It is here called "water-worth." In column 5 are the percentages of the anhydrous salt which the final portions of the cryohydrates contained, according to the analyses given above. The letters M.L show that the liquids here analyzed stood before solidification in the relationship of mother-liquors to the preceding crops of cryohydrate. In column 6 are analyses of the crop of cryohydrate immediately preceding the last cryohydrate. The salts are arranged according to the degree of cold attainable when the salt is used as a cryogen, *i. e.* in a freezing-mixture.

TABLE X.—Showing (1) the chemical formula of the salt, (2) the lowest temperature to be got by mixing the salt with ice, (3) temperature of solidification of the cryohydrate, (4) molecular ratio between anhydrous salt and water of its cryohydrate (water-worth or aquavalent), (5) percentage of anhydrous salt in portion of cryohydrate last to solidify, (6) percentage of anhydrous salt in crop of cryohydrate before the last.

(1) Formula of salt.	(2) Temperature of cryogen.	(3) Temperature of solidification of cryohydrate.	(4) Molecular ratio or water-worth or aquavalent.	(5) Percentage of anhydrous salt in last cryohydrate. M.L.	(6) Percentage of anhydrous salt in next to last cryohydrate.
$NaBr$	−28°	−24°	8·1	41·33	41·61
NH_4I	−27	−27·5	6·4	55·49	57·6
NaI	−26·5	−15	5·8	59·45	59·39
KI	−22	−22	8·5	52·07	51·72
$NaCl$	−22	−22	10·5	23·60	
$SrCl_2+6H_2O$	−18	−17	22·9	27·57	27·5
$NH_4{}^2SO_4$	−17·5	−17	10·2	41·70	42·2
NH_4Br	−17	−17	11·1	32·12	32·17
NH_4NO_3	−17	−17·2	5·72	43·71	43·26
$NaNO_3$	−16·5	−17·5	8·13	40·80	41·3
NH_4Cl	−16	−15	12·4	19·27	19·27
KBr	−13	−13	13·94	32·15	31·80
KCl	−10·5	−11·4	16·61	20·03	20·07
K_2CrO_4	−10·2	−12	18·8	36·27	36·41
$BaCl_2+2H_2O$	− 7·2	− 8	37·8	23·2	24·0
Sr_2NO_3	− 6	− 6	33·5	25·99	25·91
$MgSO_4+7H_2O$	− 5·3	− 5	23·8	21·86	
$ZnSO_4+7H_2O$	− 5	− 7	20·0	30·84	
KNO_3	− 3	− 2·6	44·6	11·20	
Na_2CO_3	− 2·2	− 2	92·75	5·97	
$CuSO_4+5H_2O$	− 2	− 2	43·7	16·89	
$FeSO_4+7H_2O$	− 1·7	− 2·2	41·41	16·92	17·35
K_2SO_4	− 1·5	− 1·2	114·2	7·80	7·5
$K_2Cr_2O_7$	− 1	− 1	292·0	5·30	
$Ba2NO_3$	− 0·9	− 0·8	259·0	5·30	
$Na_2SO_4+10H_2O$	− 0·7	− 0·7	165·6	4·55	
$KClO_3$	− 0·7	− 0·5	222·0	2·93	2·88
$Al_2NH_42SO_4+12H_2O$	− 0·4	− 0·2	261·4	4·7	4·2
$HgCl_2$	− 0·2	− 0·2	450·0	3·24	3·29

§ 89. *Remarks on Table*—The above Table contains the whole of the salts which I have as yet examined fully. The interesting group of the chlorides of the alkaline earths, including magnesium and the no less interesting group of the perchlorides of aluminium and iron, have presented difficulties with which I am still contending. The same is the case with the nitrate of calcium and the chloride of copper, $Cu\,Cl_2$.

From the evidence before us I think, however, that I may venture to enunciate the general law, that if we define as similar salts either (1) those which consist of the same acid united with bases belonging to the same chemical group (ex. Na_2SO_4, K_2SO_4), or (2) those which consist of the same base united with acids belonging to the same group (ex. KNO_3, $KClO_3$), or (3) those whose bases belong to the same group, and whose acids belong to the same group—then, *of similar salts, the one which produces the greatest cold when used in a freezing-mixture unites as a cryohydrate with the fewest molecules of water*. And to the following law there seems to be only one pronounced exception: *The temperature at which the cryohydrate is formed is the same as the temperature of the corresponding freezing-mixture.* This latter law, however, has to be taken with reserve as far as those salts are concerned which, like $AlCl_3$ and $MgCl_2$, decompose water, and also in regard to those bodies which, like $CaCl_2$, unite with water under the liberation of much heat. These I shall consider in my next communication to the Society.

Cryohydrate of Ethylic Alcohol.

§ 90. Of very great interest is the behaviour which is shown by mixtures of ethylic alcohol and water when deprived of heat. This interest extends itself in a practical direction, in consequence of the use of alcoholic liquids in regions of extreme cold. We have here at once a new element for consideration. The two liquids are miscible in all proportions. This means that any possible hydrate of alcohol is soluble at ordinary temperatures both in water and in alcohol. I shall use the word alcohol to denote absolute alcohol, C_2H_6O, and the word "spirit" for a mixture of this with water.

§ 91. The fact so long known, that heat is liberated and volume finally lost when ethylic alcohol is mixed with water, has silently pointed to the conclusion that there must be at least one definite hydrate of alcohol. It is sufficiently clear that if one were forced to the alternative of relying either upon the amount of heat liberated or upon the loss of volume, the former rather than the latter would be the most trustworthy.

§ 92. *Historical.*—A useful historical summary of much of what has been previously done in France in this direction of research is prefixed to a recent paper on the subject by M. Melsens, in the *Annales de Chimie et de Physique*, entitled "Sur la refroidissement et la congélation des liquides alcoholiques et des vins." According to M. Boussingault, frozen wines after thawing furnish an alcoholic liquid and are not therefore pure ice. According to M. Melsens, alcoholic liquids containing about 50 per cent. of alcohol by weight or by volume become at -30° C. viscid, syrupy, and sometimes opalescent. These represent commercial spirits such as rum, cognac, &c., and may be represented by the formula $C_2H_6O + 3H_2O$, corresponding to the maximum condensation. According, again, to M. Melsens, when wine which has become semisolid through being exposed to cold of a freezing-mixture is drained through wire gauze or introduced into a turbine, nearly colourless ice free from alcohol is left. From frozen wine containing from 10 to 12 per cent. of alcohol, from 16 to 25 per cent. of pure ice was got by means of a screw press. By the same means a frozen red or white Burgundy yielded 40 per cent. of ice.

§ 93. In the first of a series of able researches on the physical properties of mixtures of water with the alcohols, Messrs. Dupré and Page (Proc. Roy. Soc. March 11, 1869) examined, amongst others, the quantity of heat developed on mixing alcohol and water in various proportions, the specific heat of such mixtures, their capillarity, boiling-point, and their compressibility.

The following fragments of these experimenters' Tables include the critical values.

Table showing number of heat-units evolved from 5 grammes of mixtures resulting from mixing the percentages by weight of alcohol in column 1 with the complementary percentage of water. The asterisk shows the critical region.

per cent. alcohol.	Heat-units from 5 grms. of mixture.
10	26·68
20	43·95
30	47·98*
40	44·86
45	38·81

In the next Table Messrs. Dupré and Page's numbers are given, showing the specific heats of such mixtures. The column 1 shows the percentage of alcohol, column 2 the specific heat, column Δ the difference between the observed and calculated specific heats.

Alcohol per cent.	Specific heat.	Excess over calculated, Δ.
10	103·58	+ 7·53
20	104·36	12·27
30	102·60	14·47*
40	96·80	12·63
45	94·19	12·00
50	90·63	10·42

Messrs. Dupré and Page epitomize one branch of their research as follows:—"The whole of the physical characters of mixtures of alcohol and water come to a maximum deviation from their theoretical values somewhere between 30 and 45 per cent. of alcohol by weight. The 30 per cent. nearly corresponds to the formula $C_2H_6O + 6H_2O$ (29·87 per cent.); the 45 per cent. has approximately the formula $C_2H_6O + 3H_2O$ (46 per cent.)." The mean of these values is

$$C_2H_6O + 4{\cdot}5H_2O.$$

§ 94. According to Rudberg (Pogg. *Ann.* vol. xiii. p. 496), the contraction is greatest when 55 volumes of alcohol are mixed with 45 volumes of water, or 43·6 weights of alcohol with 45 weights of water. This corresponds to the formula

$$C_2H_6O + 2{\cdot}6H_2O.$$

§ 95. According to Bussy, alcohol not stronger than 33 Beaumé may be frozen by the evaporation of SO_2. This strength is that of 78·29 per cent. of alcohol by weight, or

$$C_2H_6O + 1{\cdot}406\,H_2O.$$

According to Marchand (*Journ. für Chemie*, vol. xxv. p. 253), when 1 part by weight of spirit is mixed with 1 part of snow, the depression of temperature depends upon the strength of the spirit, according to the following Table:—

per cent. by weight.	
90	−22°
70	−21
60	−19
50	−17·5
40	−16
30	−13
20	− 8

This is, I believe, the condition of the question as left by others.

§ 96. *My own experiments.*—The alcohol I used was shaken with dry carbonate of potassium and distilled from quicklime. With this I made decimal mixtures ranging from 95 alcohol and 5 of water to 95 water and 5 of alcohol. These were submitted in turn to the action of a cryogen, in order first of all to see at what temperature the solidification of each mixture *begins*. For if, as has been supposed, and quite recently by Melsens again proved, a very weak (10 per cent.) spirit gives up only ice, and since alcohol is notoriously not to be solidified by our most powerful cryogens, it must follow that during the continuous solidification of a weak spirit the temperature must fall continually. Temperatures down to −19° were observed on the mercurial thermometer. For lower temperatures, demanding the employment of solid carbonic acid and ether, use was made of an alcohol thermometer, which was collated with the mercurial one at −22°. The Table gives the temperature at which the solution began to yield solid matter. What this solid matter consists of I have afterwards to consider. Columns Δ_1 and Δ_2 are the values of the first and second differences respectively.

Table XI.—Temperatures at which Solidification begins in Spirits of various strengths.

Water per cent., by weight.	Alcohol per cent., by weight.	Temperature C. at which solidification begins.	Δ_1.	Δ_2.
95	5	− 2°	2·3	+0·6
90	10	− 4·3	2·9	+0·6
85	15	− 7·2	3·5	+0·5
80	20	−10·7	4·0	+0·7
75	25	−14·7	4·7	−0·8
70	30	−19·4	3·9	−0·2
65	35	−23·3	3·7	+0·3
60	40	−27	4·0	+2·0
55	45	−31	6·0	−1·0
50	50	−37	5·0	−2·0
45	55	−42	3·0	+5·0
40	60	−45	8·0	?
35	65	−53	?	
30	70	−65 (not).		

From this Table it is seen that the temperature of initial solidification sinks so regularly that the column of second differences shows for a long time the value 0·6. Only at lower temperatures, which of course cannot pretend to the same degree of accuracy, are serious variations visible. There is a rapid fall at the ratio 35 water to 65 alcohol; and at the ratio 30 water to 70 alcohol, I failed to effect solidification at −65° C.

§ 97. So free from discontinuity are these numbers, that one might be readily misled into the belief that the solid matter separated is in all cases ice. This, however, is not the case. Down to the ratio 65 water and 35 alcohol, the ice spicula freely moving in the mother-liquid are easily recognized. If they are remelted they invariably reform at the same temperature. At ratios of 60 water to 40 alcohol and stronger, the aspect of the solid formed is quite different. The liquid may then acquire the consistency of Canada balsam, and yet, if kept still, it may remain perfectly transparent. On rubbing with the thermometer-bulb or with a glass rod minute crystals are formed, the liquid becomes more mobile, resembling old honey. It offers a notable case of a condition of supersaturation producing a temporary colloid condition.

§ 98. Finding that this phenomenon was first noticeable at about the solution of the ratio 60 water to 40 alcohol, and reflecting that this is nearly the ratio of one molecule of alcohol to four of water, I made a spirit of exactly this molecular ratio, namely 39·07 alcohol and 60·93 of water by weight. On submitting this to the action of a cryogen, I found the remarkable result that nothing separated till the temperature −34° was reached, although both weaker and stronger solutions begin to solidify at higher temperatures. The solid formed in this case is perfectly white and opaque, and the temperature remains constant till the whole has become perfectly dry. This pearly aspect and the constancy of temperature throughout the solidification betray the cryohydrate. Both weaker and stronger spirits sink in temperature as they solidify—the former until by the elimination of ice it presents a mother-liquor of the 4-hydrate composition, the latter by the elimination of liquid alcohol as the solid 4-hydrate is formed.

§ 99. When, therefore, a dilute alcohol (say 95 water to 5 alcohol) is cooled, ice separates out, the spirit becomes stronger and stronger, and the temperature lower and lower. When the ratio $C_2H_6O + 4H_2O$ is reached the temperature −34° C. is reached, and the remainder of the solution is a cryohydrate of alcohol whose melting- and freezing-point is −34°, and whose composition is $C_2H_6O + 4H_2O$.

On the other hand, if a spirit a little stronger than $C_2H_6O + 4H_2O$ is subjected to cold, nothing separates till about −27° C. At this temperature the cryohydrate $C_2H_6O + 4H_2O$ begins to separate out, and as it relinquishes the alcohol the solidifying-point of the mother-liquor falls. Observe, the cryohydrate separates from alcohol at a temperature above its own melting-point when alone. It follows from this that solidification may commence at the same temperature in two spirits of different strengths, provided they are both close about the 4-molecule hydrate in composition. But while in the weaker the solid will be ice, in the stronger it will be the cryohydrate. The apparent anomaly arises from the circumstance that the cryohydrate is soluble both in water and alcohol, and that ice is soluble in the liquid cryohydrate. We are here reminded of the phenomenon of the maximum density of water, which I have already endeavoured to connect with the formation of a cryohydrate of water (in § 36).

§ 100. For some reasons this melted cryohydrate of alcohol, or spirit of wine containing 60·93 water and 39·07 of alcohol, should be the standard or proof spirit in all alcoholometry.

English proof spirit contains 50·5 per cent. by weight of alcohol. It accordingly should begin to solidify at −38° (about the freezing-point of mercury). At lower temperatures it becomes a pasty mass, but never perfectly brittle, on account of the unfreezable alcohol in excess above that of the cryohydrate. Spirits of 39 per cent. and under become perfectly solid before the temperature reaches −36°. The very fact that weaker spirits wholly solidify while stronger never do so is, I conceive, a complete proof of the existence of a cryohydrate. For if under all circumstances water alone solidified, alcohol would be left even from the weakest spirits, and total solidity could never be reached.

§ 101. The above considerations, of course, only apply to the chemically pure substances alcohol and water; how far the sugar, caramel, fusel, essences &c. of commercial spirits, and the innumerable substances in wines and beers may affect the solidification is a matter for future research. It is quite possible, for instance, that in some rums the 10 per cent. alcohol above that required for the cryohydrate might find sufficient foreign matter present to form therewith a solid, and so allow the whole to solidify.

§ 102. I believe that the detection of this 4-molecule hydrate of alcohol reconciles the apparently contradictory results of former experimenters. Thus, looking on wine as a 10-per-cent. spirit of wine, M. Melsens obtained by a freezing-mixture 40 per cent. of ice. I judge from Table XI. that the temperature reached was −8°, if we make no allowance for loss of ice during manipulation.

§ 103. The evidence adduced by Messrs. Dupré, Page, and others points to the existence of at least one hydrate of alcohol; but I am not prepared to say that such hydrate is necessarily identical with the 4-molecule cryohydrate which we have been considering. It must be remembered in this connexion that the cryohydrates of solid salts contain more water than any other known hydrate; and the existence of the 4-molecule cryohydrate rather, I conceive, tends to show that the ordinary hydrate of alcohol contains less water, and is, as some of Dupré and others' experiments may be interpreted as showing, the 3-hydrate

$$C_2H_6O+3H_2O.$$

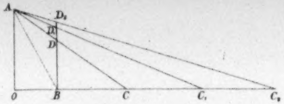

Fig. 1.

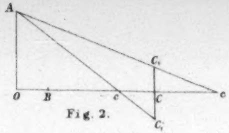

Fig. 2.

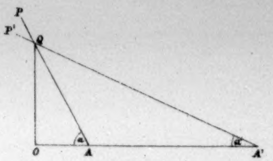

Fig. 3.

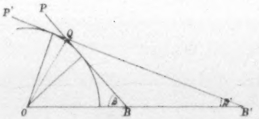

Fig. 4.

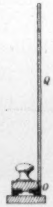

Fig. 5a.

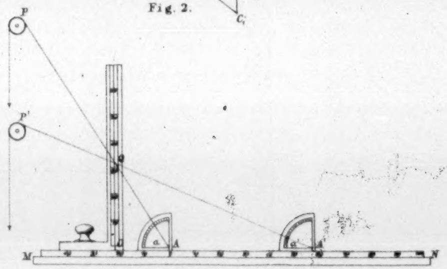

Fig. 5.

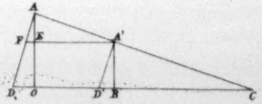

Fig. 6.

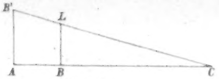

Fig. 7.

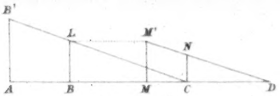

Fig. 8.

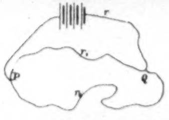

Fig. 9.

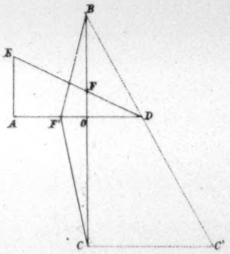

Fig. 10.

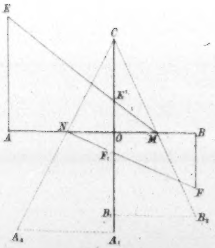

Fig. 11.

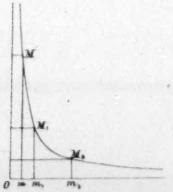

Fig. 12.

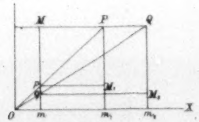

Fig. 13.

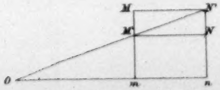

Fig. 14.

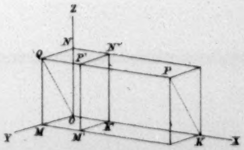

Fig. 15.

Cryohydrate of Ether.

§ 104. *Cryohydrate of Ether* (?).—It is well known that when water and ethylic ether are shaken together mutual but only partial mixture ensues, the water taking up about one ninth of its volume of ether, and the ether about one thirty-sixth of its volume of water. Such an aqueous solution of ether when subjected to cold, solidifies at −2° C., without any separation of ether, into a dry solid consisting of ether and water. The compound exhibits an interesting feature, inasmuch as it exemplifies the effect of cooling on the luminosity of flame. The cryohydrate of ether may be solidified in a long test-tube, and thence removed, presenting the appearance of a candle. One end of this is cut off flat, and the whole placed upright on a plate. A light being applied at the top, the whole melts away as the ether burns. The flame is quite non-luminous. The ether is only free to burn as it is in the act of melting ice, and is so cooled. So it is well known that marsh-gas becomes luminous if heated before combustion. Thus ether (which is empirically olefiant gas plus water) and alcohol (which is empirically ether plus water) have flames of luminosity the less according as their ratio of potential water is greater. The cryohydrate of ether is in fact physically, as far as its luminosity is concerned, an alcohol.

Throughout this second part of this research I have been much indebted to my friend Mr. R. Cowper for his very zealous and skilful assistance.

XIV. *On Graphical Methods of solving certain simple Electrical Problems.* *By* Prof. G. Carey Foster, *F.R.S.*[*]

[With a Plate.]

It is probably very seldom that the geometrical representation of the mathematical relations between physical quantities is as convenient, for purposes of investigation, as the corresponding analytical expression; but for purposes of exposition, as distinguished from those of investigation, geometrical constructions often possess considerable value. In particular, whenever they can be put into a simple form, their physical significance is more easily understood, than that of an algebraical formula, by those who are but little accustomed to mathematical modes of expression; and they generally exhibit with greater clearness the effect of a variation of any one of the related magnitudes. I have therefore thought that it might be worth while, for the sake of trying to draw increased attention to the utility of geometrical methods in elementary physical teaching, to point out in detail their application to a few important electrical problems, such as some of the simplest cases which come under Ohm's law of the relations between electromotive force, strength of current, and resistance.

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[*] Read November 7, 1874.

If the equation which expresses Ohm's law be written in the three following ways—

$$\frac{E}{C}=R, \qquad \frac{E}{R}=C, \qquad CR=E,$$

where E stands for *electromotive force*, C for *strength of current*, and R for *resistance*, we have at once suggested three different modes of geometrical construction by means of rectangular coordinates, the coordinates representing in the three cases respectively (1) electromotive force and strength of current, (2) electromotive force and resistance, and (3) strength of current and resistance.

I. *Ordinates represent Electromotive Forces, and Abscissæ represent Strengths of Current.*

This system, according to which *resistance* comes to be represented by the *slope* of a straight line (that is, by the tangent of the angle which the line makes with the axis of abscissæ), was lately employed by M. Crova* for the discussion of experiments relating to the degree of constancy possessed by so-called "constant" galvanic batteries; and its application to several other problems, including some of those treated by other methods in this paper, has been still more recently† pointed out by the same author. It is therefore not needful to discuss it further in this place.

* *Comptes Rendus*, 6th April 1874, vol. lxxviii. p. 965.

† *Journal de Physique* (Sept. 1874), vol. iii. p. 278.

II. *Ordinates represent Electromotive Forces, and Abscissæ represent Resistances.*

This system was used long ago by Ohm*, and has been frequently employed since his time, though perhaps chiefly by practical electricians†. The following examples may serve to illustrate its application to questions connected with galvanic circuits in which there is a constant electromotive force.

Let O A (Plate III. fig. 1) represent the electromotive force of a battery, O B the resistance of the battery, and B C the resistance of the remainder of the circuit, this being made up of simple metallic conductors in which no additional electromotive force acts, then the slope of the straight line A C, or the *tangent* of the angle A C O, represents the strength of the current. It is obvious, by a glance at the figure, that the strongest current that the given battery could produce would be obtained by making the external resistance B C equal to nothing, and that it would be represented by the slope of the line A B, or by tan $\angle$ A B O. Also it is evident that, if the external resistance is increased by equal amounts $C C_1, C_1 C_2, \ldots$ each equal to B C, the strength of the current, as denoted by the slope of the line drawn from A to the points $C, C_1, C_2 \ldots$, diminishes by smaller and smaller amounts for each equal increment of resistance, and that it would not vanish for any finite value of the resistance.

If any electromotive force acts in the part of the circuit external to the battery, its effect on the strength of the current can be represented by drawing through C a line $C C'$ parallel to O A, and of length proportional to the external electromotive force, *upwards* if this electromotive force is *inverse*, *downwards* if it is *direct*,—and drawing the straight line $A C'$ (fig. 2). If c be the point where this line cuts O C, tan $\angle$ AcO measures the strength of the current. Of course the effect of any electromotive force outside the battery could also be represented by a diagram such as fig. 1, if the line O A were there taken to represent, not the electromotive force of the battery, but the total resultant electromotive force of the whole circuit.

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* *Die galvanische Kette mathematisch bearbeitet*, 1827.

† See, for example, R. S. Culley, 'Handbook of Practical Telegraphy,' 1874, pp. 352-355; Latimer Clark, 'Elementary Treatise on Electrical Measurements,' 1868; G. K. Winter, "On Earth-Currents," Journal of the Society of Telegraph-Engineers, vol. ii. pp. 89-101; &c.

If a line be drawn from B (fig. 1) parallel to O A, the length B D, B D_1, B D_2 . . . intercepted by the straight line through A, whose slope gives the strength of the current, represents the difference of potential between the terminals of the battery, or, in other words, the electromotive force which is effective in maintaining a current in the external conductor. The figure shows that this varies between a maximum (=O A, the total electromotive force of the battery) when the external resistance is infinite (contact broken) and a minimum (=0) when the external resistance is nothing. If two values, B D and B D_1, of the externally effective electromotive force are known, which correspond respectively to two known values B C and B C_1 of the external resistance, it is evident that the electromotive force and internal resistance of the battery will be given by drawing the straight lines C D and C_1 D_1, producing them till they meet in A, and letting fall from A a perpendicular A O on C B produced: A O and O B then represent respectively the values required. Experimentally, the values to be given to B D and B D_1 could be found by direct measurement with an electrometer; or they could be got from the relation $e' = cr'$, where e' is the externally effective electromotive force and c the current as measured by a galvanometer in a circuit of external resistance $= r'$.

From the above relations it is easy to deduce a construction, which may sometimes be of practical use, *for finding the permanent resistance and electromotive force of a constant battery from two deflections of a galvanometer without using trigonometrical tables.* This construction requires to be slightly modified according to whether the instrument used is a tangent-galvanometer or a sine-galvanometer. It is as follows:—

1. *For a tangent-galvanometer.*—The battery is connected in simple circuit with a tangent-galvanometer, and the deflection α of the galvanometer is observed; then a known resistance r' is added to the circuit and the deflection is observed again. Let the second deflection be denoted by α'. The following construction then gives the electromotive force of the battery ($=e$) and the permanent resistance of the circuit ($=r$).

From any point A in the straight line O A (fig. 3) draw A P, making the angle O A P $=\alpha$; produce O A to A$'$, making A A$'$ proportional to the added resistance r', and from A$'$ draw A$'$ P$'$, making the angle O A$'$ P$'=\alpha'$ and on the same side of O A as A P. Since α' is less than α, the straight lines A P and A$'$ P$'$ will intersect. From the point of intersection Q draw Q O perpendicular to O A. Then O A represents the permanent resistance of the circuit, and Q O represents the electromotive force, in terms of that electromotive force taken as unity which, if it acted in a circuit of unit resistance, would generate a current capable of causing a deflection of 45° on the particular galvanometer employed.

2. *For a sine-galvanometer.*—Let β be the deflection observed when there is no extra resistance, and β' the deflection when an additional resistance r' has been inserted in the circuit. Make B B$'$ (fig. 4) proportional to r', make the angles O B P and O B$'$ P$'$, on the same side of B B$'$, equal respectively to β and β', and let B P and B$'$ P$'$ intersect at the point Q. Draw Q O to bisect the external angle B Q P$'$, and cutting B$'$ B produced in O; then O B represents the permanent resistance of the circuit, while the radius of a circle drawn with the point O as centre so as to touch the straight lines B P and B$'$ P$'$ measures the electromotive force, the unit of measurement for the latter being the electromotive force which, in a circuit of unit resistance, would give a current strong enough to deflect the galvanometer used through 90°*.

If the constructions indicated above are carried out for several different values of the external resistance, it is clear that, with a strictly constant battery, lines drawn according to the same rule as A P and A$'$ P$'$, in the case of a tangent-galvanometer, will all pass through the same point, and that, in the case of a sine-galvanometer, all lines drawn in the same way as B P and B$'$ P$'$ will be tangents to the same circle.

These constructions are so simple and can be so quickly made with sufficient accuracy, that by means of them the effect of altering the resistance of a circuit or the number or arrangement of the cells of a battery can be exhibited to a class by the help of actual measurements made during a lecture; but in order to make the process still more rapid, I have had an apparatus made, which may be called "A Galvanometric Slide-Rule," whereby, when two deflections of a galvanometer have been observed corresponding to a known difference of resistance, the permanent resistance and electromotive force of a battery can be ascertained in the course of a few seconds. The general arrangement of this apparatus is shown in fig. 5, which is drawn to a scale of about $\frac{1}{20}$. M N is a wooden base about 155 centims. long by 15·2 centims. wide, and 4·7 thick, with a groove in the upper surface, of the shape shown in fig. 5*a*, running from end to end. On the vertical side shown in the figure there is a scale 150 centims. long, divided into millimetres and numbered towards right and left from zero at a point A 50 centims. from M and 100 centims. from N. Exactly above the zero mark of the scale is a small brass stud, the axis of which passes through the centre of a small graduated quadrant of 12 centims. radius. Another, similar quadrant, A′, with a brass stud at the centre is attached to a small board which slides on the upper side of M N and can be clamped by a screw in any position between A and N. There is also a vertical scale 65 centims. long, divided into millimetres and numbered from the bottom upwards, which is fastened to a sliding piece C, whereby it can be clamped in any position between M and A. The faces of the vertical scale and of the quadrants A and A′ are flush with the face of the horizontal scale M N. Two thin silk cords, stretched by small weights, are passed over pulleys at P and P′, and attached one to each of the pins at the centres of the quadrants A and A′. The pulleys are so placed that the cords are very nearly in the same vertical plane as the scales M N and C Q. The apparatus is used as follows:—The pulley P is raised or lowered until the cord fixed at A shows, upon the corresponding quadrant, the deflection obtained on the tangent-galvanometer when no extra resistance is added to the circuit; the sliding piece carrying the quadrant A′ is then moved towards right or left so that the number of centimetres in the distance A A′ may be the same as the number of units of resistance (or, if more convenient, so that it may be a simple multiple or submultiple of this number) added to the circuit in order to get a second reading of the galvanometer; then the pulley P′ is adjusted so that the cord passing over it may indicate upon the quadrant B the galvanometer-deflection obtained after introducing the resistance corresponding to A A′; and lastly the vertical scale is moved so that a vertical line drawn through the centre of the division-marks and continued downwards to meet the horizontal scale at O may be exactly behind the point Q, where the cords A P and A′ P′ cross each other. We have then only to read off the horizontal and vertical distances, A O and O Q, to get the permanent resistance and electromotive force of the circuit*. To adapt the apparatus to use with a *sine-galvanometer*, it would be only necessary to replace the vertical scale O Q by a quadrant marked with concentric circular arcs, each differing from the next by 1 millim. in radius.

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* It may be worth while to point out that any galvanometer may be used as a sine-galvanometer, even though it is not provided with a graduation to show the angle through which it has been turned. It is only needful, after setting the instrument so that the zero of the scale is exactly below the needle while the current is passing, to interrupt the current; the needle then swings away from the zero-mark, returning to the magnetic meridian; and the angle now indicated by it is the angle through which the galvanometer has been turned from the meridian.

The following additional examples may be given of the same mode of treatment in connexion with allied problems :—

Development of Heat in a Galvanic Circuit.—Draw O C (fig. 6) horizontally to represent the resistance of the circuit, and O A vertically to represent the electromotive force; from A draw A C and also A D at right angles to A C; then O D represents the heat produced in the circuit per unit of time. It is of course to be understood that O A denotes not necessarily the electromotive force of the battery, but the resultant electromotive force of the whole circuit—that is, the algebraic sum of all the electromotive forces which act anywhere in it. If O B denote the resistance of the battery and B C the remaining resistance of the circuit, and if B A′ be drawn vertically, A′ E F horizontally, and A′ D′ at right angles to A C, E F will denote the heat generated in the battery, and B D′ the heat generated in the external part of the circuit.

Strength of Currents in the different branches of a divided Circuit.—In considering this problem it is needful first of all to have a mode of representing geometrically *the combined resistance of two or more conductors connected in "multiple arc."* Let A B and B C in the same straight line (fig. 7) represent the resistances of two conductors taken separately. Draw A B′ equal to A B and at right angles to it; join B′ C, and through B draw a line parallel to A B′ and cutting B′ C in L; then B L represents the combined resistance of the two conductors. The same process is applicable to any number of conductors. Thus, let A B, B C, and C D (fig. 8) represent the separate resistances of three conductors connected in multiple arc. Making the construction indicated above, B L represents the resistance equivalent to A B and B C in multiple arc. Draw L M′ parallel to A B C D, make C M = B L, draw M M′ at right angles to B C, join M′ D, and draw C N perpendicular to C D. C N then represents the joint resistance of the three conductors.

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* The quadrant A is made so that it can be turned aside to allow of the vertical scale being brought close up to the zero-point of the horizontal scale when low battery-resistances have to be measured.

Now consider a conducting-system such as that indicated in fig. 9, where two points, P and Q, are connected through three conductors whose resistances are respectively r, r_1, and r_2; and let a battery of electromotive force e make part of the first conductor. The strength of the currents and the distribution of potential in the various parts of the system can be represented as follows:—Take O A (fig. 10) to represent r, from O draw O B and O C in opposite directions perpendicular to O A, make C C′ equal to O C and perpendicular to it, draw B C′ cutting A O produced in D; then O D represents the joint resistance of r_1 and r_2. Draw A E perpendicular to A O to represent the electromotive force e, and join E D. Then tan ∠ A D E represents the strength of the current in the battery and the sum of the currents in the two parallel branches. These may be obtained separately thus: let the point of intersection of E D and O B be denoted by F; then O F represents the difference of potential between the points P and Q or the electromotive force which is effective in the two conductors of resistance r_1 and r_2. In O A make O F′ = O F and draw F′ B and F′ C; we have then, for the strength of the currents in the conductors whose resistances are represented by O B and O C, tan ∠O B F′ and tan ∠O C F′ respectively.

Next let *two* of the conductors connected together at P and Q contain galvanic batteries, and, as before, let the resistance of the branch containing one battery be r, while that of the branch containing the other is r_1, and let the electromotive forces be e and e_1 respectively. If the batteries are so connected that both tend to make the potential at P differ from that at Q in the same sense, we have an arrangement of which a special case is presented by Poggendorff's "compensation method" for the comparison of electromotive forces. To obtain a geometrical expression for the strengths of the currents in the various parts of the circuit in the general case (that is, without assuming that there is "compensation" in any branch), we may proceed in the following manner:—Take O A and O B in the same straight line (fig. 11) to represent the resistances r and r_1 of the two branches including the batteries, and O C at right angles to A B to represent the resistance r_2 of the third branch. In C O produced make $OA_1 = OA$, and $OB_1 = OB$, also draw $A_1 A_2$ equal and parallel to O A and $B_1 B_2$ equal and parallel to O B, and join A_2 C and B_2 C. Let A_2 C cut O A in N, and let B_2 C cut O B in M. From A draw A E at right angles to A O to represent the electromotive force e, and draw E M cutting O C in E′; similarly, from B draw B F* to represent the electromotive force e_1, and draw F N cutting C O (produced) in F′. Then $F'E' = F'O + OE'$ represents the electromotive force which is effective in the conductor of resistance r_2 represented by O C, and $AE - F'E'$ and $FB - F'E'$ represent the electromotive forces which are effective in the branches of resistance r and r_1 respectively.

III. *Ordinates represent Strength of Current, and Abscissæ represent Resistances.*

With this system of coordinates, electromotive force is expressed by the area of a rectangle. Thus, if a given battery produces a current whose strength is represented by the ordinate M m of the point M (fig. 12), in a circuit the resistance of which is represented by the abscissa O m of the same point, its electromotive force must be proportional to the area of the rectangle O M; and if the battery is "constant," the currents, represented by the ordinates $M_1 m_1$, $M_2 m_2$, and corresponding to the resistances denoted by the abscissæ $O m_1$, $O m_2$, will be such that the areas of the rectangles O M, O M_1, and O M_2 are all equal; and hence the characteristic property of the battery will be expressed by the curve which is the locus of the points M, M_1, &c.—that is to say, by a rectangular hyperbola whose asymptotes are the axes of no current and no resistance. When the hyperbola characteristic of a given battery is drawn, it is of course easy by measuring coordinates to find what current would flow through a given resistance, or conversely to find what must be the resistance of the circuit in order that the current may have a given strength; but the difficulty of tracing an hyperbola with accuracy greatly lessens the practical utility of this method of calculation. Since, however, the asymptotes are fixed, each hyperbola is completely defined when one point of it is given; and, in like manner, when the corresponding values of current and resistance are known in any one case for a given battery, each is determined for any other case when the value of the other is given. Accordingly the actual drawing of an hyperbola is not necessary; for when one point is assigned, any other points corresponding to given problems can be easily found.

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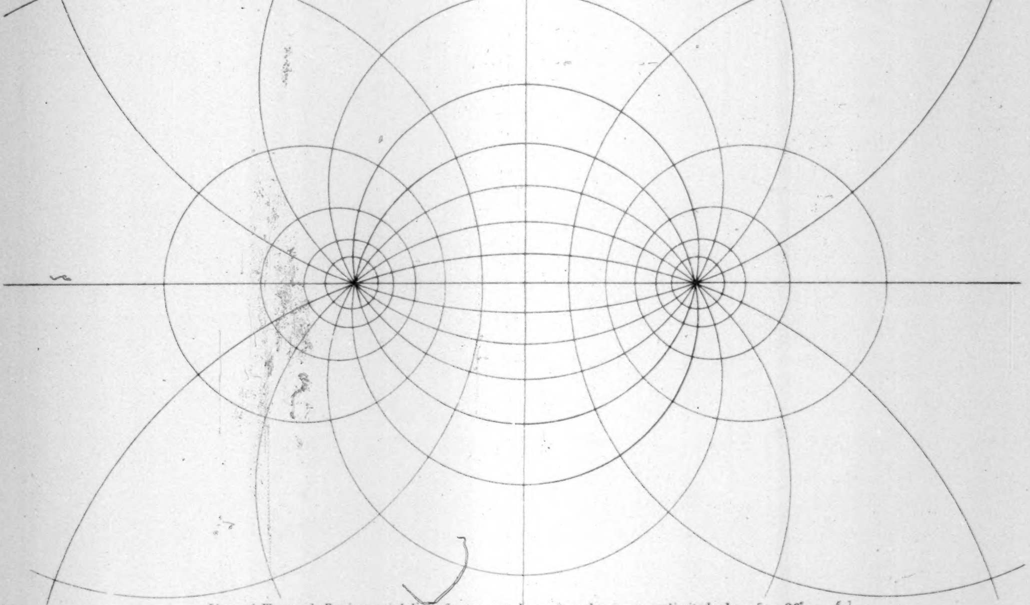
* B F must be drawn in the *opposite* direction to A E if, as supposed above, the batteries are so connected that the difference of potential between the points P and Q due to each battery separately is of the *same* sign.

For instance, let the coordinates of the point M (fig. 13), referred to the axes O X and O Y, represent respectively the resistance of a conductor and the strength of the current produced in it by a given electromotive force, and let it be required to find the strength of the currents which the same electromotive force would generate in conductors whose resistances are respectively represented by the abscissæ O m_1 and O m_2. Through M draw a line M P Q parallel to O X, and through m_1 and m_2 draw m_1 P and m_2 Q parallel to O Y; join O P and O Q, and let p and q be the points in which O P and O Q respectively cut M m; then mp and mq will represent the strengths of the required currents; and if lines be drawn parallel to O X through p so as to intersect m_1 P in M_1, and through q so as to intersect m_2 Q in M_2, M_1 and M_2 will be points whose coordinates, like those of M, represent corresponding values of resistance and strength of current. The points M, M_1, and M_2, therefore, lie upon the same rectangular hyperbola.

In a similar way we may treat many problems of the same sort as those discussed above by aid of what for distinction may be called Ohm's construction; but as the constructions, arising from the choice of resistance and strength of current as coordinates are usually rather more complex than those previously given, and as I have not come across any cases in which they appear to be decidedly more expressive, I will only give two additional examples.

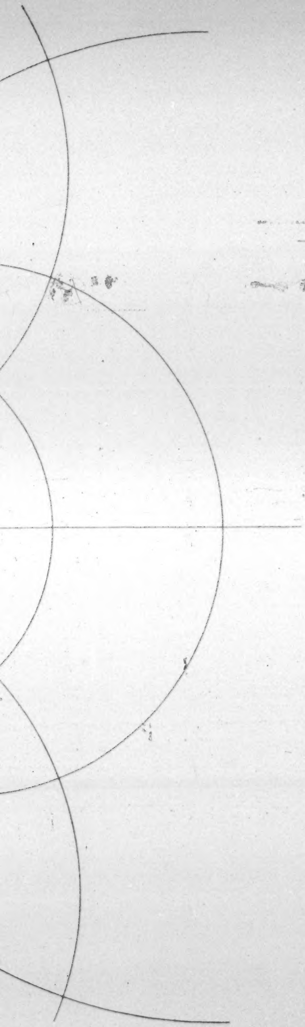
To find the permanent resistance and the electromotive force of a battery from observations of the strengths of two currents corresponding to resistances which differ by a known amount.—Let $m\,M$ (fig. 14) represent the strength of the current when the resistance of the circuit has an unknown value represented by the (unknown) abscissa $O\,m$; and let $n\,N$ express the strength of the current when the resistance has been increased by a known amount denoted by $m\,n$. Through M and N draw straight lines parallel to $m\,n$, and let N_1 and M_1 be the points where these lines respectively intersect $n\,N$ (produced) and $m\,M$. Draw the straight line $N_1\,M_1$ and produce it to intersect $n\,m$ produced in O; then $O\,m$ represents the original resistance of the circuit, and the rectangle on the base $O\,m$ with altitude $m\,M$, or the rectangle on the base $O\,n$ with altitude $n\,N$, represents the electromotive force.

The heat produced in unit of time by a constant current of given strength traversing a conductor of given resistance can be represented by the volume of a right square prism, two of whose dimensions represent the strength of the current, while the third represents the resistance; and in the case of a battery of constant electromotive force, the relation between the resistance of the circuit and the heat produced in unit of time can be expressed generally as follows:—Take three rectangular axes, O X, O Y, and O Z (fig. 15); in O X take O K to represent the resistance, and in O Y take O M so that the area of the rectangle K M represents the electromotive force, and therefore O M the strength of the current. Similarly, take O N in O Z also to represent the strength of the current. Then the heat generated in unit of time is proportional to the contents of the rectangular parallelopiped O P, constructed upon the lines O K, O M, and O N. The locus of P is the intersection of two equal and similar hyperbolic cylinders, whose equations are respectively $x\,y =$ constant $=$ electromotive force, and $x\,z =$ constant $=$ electromotive force, and is itself a rectangular hyperbola in the plane of O X and O Q, and having these lines for asymptotes. If $O\,K'$ represent the internal resistance of a battery, and $K'\,K$ the external resistance, the heat generated inside the battery is represented by the parallelopiped $O\,P'$, and that generated in the external conductor by the parallelopiped $K'\,P$.



Lines of Flow and Equipotential lines for two equal opposite poles in an unlimited plane [$a = 20^\circ$; $u = \frac{5}{3}$]

Proc. Phys. Soc. Vol. I. Pl. IV.



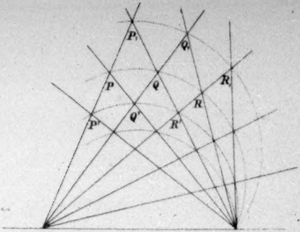


Fig. 1.

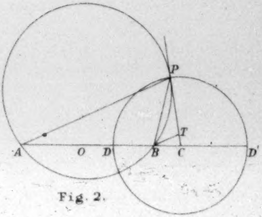


Fig. 2.

AO — OB — a
OC — d
BC — h
OD — l
OD' — l'
CP — ρ
AP — r
BP — r'

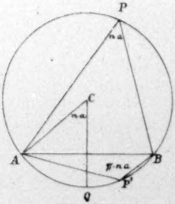


Fig. 3.

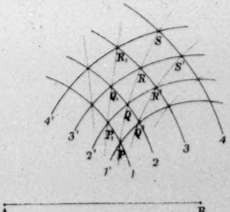
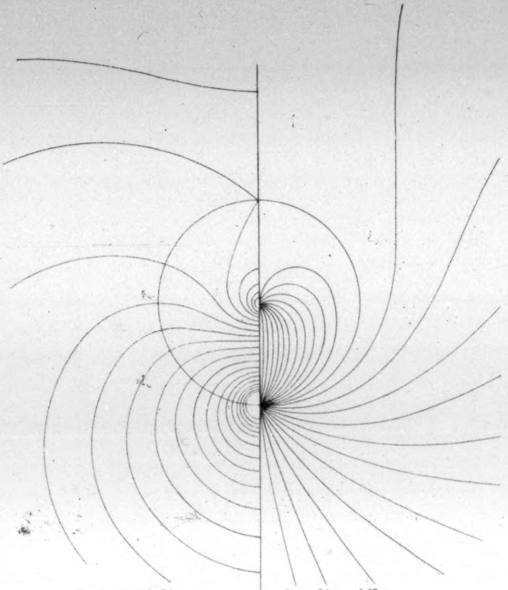


Fig. 4



Two opposite poles, one twice as strong as the other.

Fig: 6.

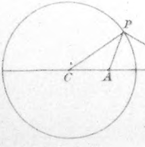


Fig. 5.

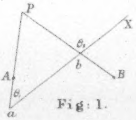
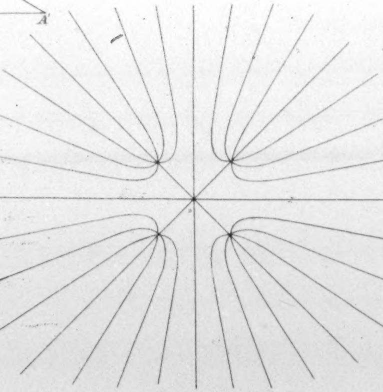
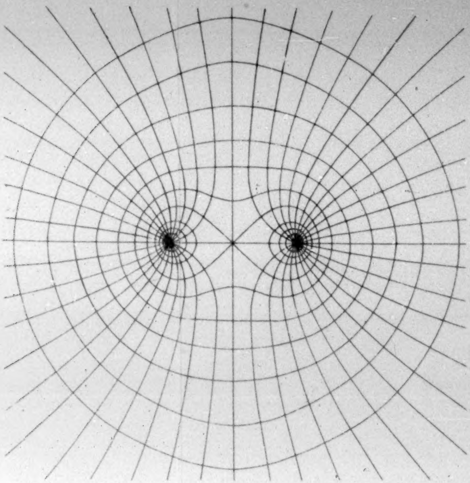


Fig: 1.



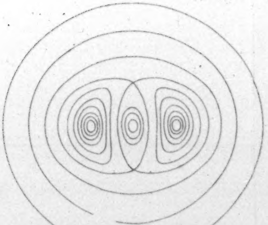
Four Equal Similar Poles.

Fig: 7.



Lines of Flow and Equipotential Lines for two equal Similar Poles. $[a = 15^\circ;\ \mu = \frac{3}{2}]$

Fig: 3.



Loci of Equal Flow for two equal Similar Poles.

Fig: 4.

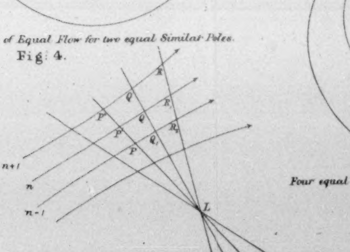
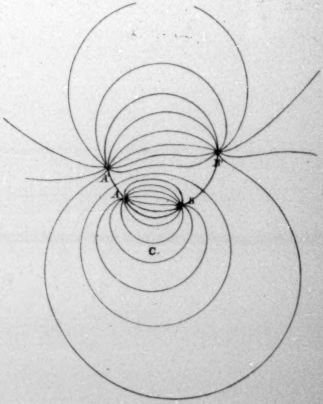


Fig: 2.



Four equal Poles: A and A' positive; B and B' negative.

Fig: 9.

PROCEEDINGS

OF

THE PHYSICAL SOCIETY

OF LONDON.

February 27, 1875.

XV. *On the Flow of Electricity in a uniform plane conducting Surface.*—Part I. *By* G. Carey Foster, *F.R.S., and* Oliver J. Lodge.

[With Two Plates.]

1. The objects of the following paper are:—first, to show that the most important laws relating to the flow of electricity in a plane conducting sheet of uniform conductivity can be established by mathematical considerations of such a simple kind that there is no reason why they should not be introduced into ordinary teaching, as well as into the more complete elementary text-books of electricity; and in the second place, to describe methods that may conveniently be employed for testing experimentally the theoretical conclusions, and to record some results obtained by these methods.

The general subject treated of in this paper has attracted the attention of a considerable number of mathematicians and physicists. The earliest published investigation relating to it is contained in a remarkable memoir by Kirchhoff, which appeared in Poggendorff's *Annalen* in 1845 (vol. lxiv. p. 497). In this paper Kirchhoff established the general mathematical theory of the flow of electricity in an unlimited uniformly conducting-sheet, and in a limited sheet with a circular boundary, with so much completeness as to leave little for others to do beyond working out the application to special cases of the general principles he laid down, or finding other methods of establishing the conclusions he deduced from them.

We cannot better indicate the general plan of Kirchhoff's investigation than by quoting the following account of it from a paper by Professor W. Robertson Smith*, to which we shall have to make further reference immediately:—"By an application of Ohm's law, he [Kirchhoff] expressed analytically the condition to be satisfied by v [the potential]. When the electricity enters and issues by a number of individual points, he found (apparently by trial) that an integral of the form $\Sigma(\alpha \log r)$, where r_1, r_2, &c. are the distances of the point (x, y) from the successive points of entrance and issue, satisfied the conditions when the plate is infinite. For a finite plate, it is necessary that the boundary of the plate should be orthogonal to the curves

$$\Sigma(\alpha \log r) = \text{const.} \quad . \quad . \quad (3)$$

He was thus led to form the orthogonal curves whose equation he gives in the form

$$\Sigma(\alpha[r, \mathrm{R}]) = \text{const.}, \quad . \quad . \quad (4)$$

where $[r, \mathrm{R}]$ is the angle between r and a fixed line R. These equations he applies to the case of a circular plate, completely determining the curves when there is one exit and one entrance point in the circumference, and showing that in any case a proper number of subsidiary points would make the equipotential lines determined by (3) cut the circumference at right angles. Kirchhoff's paper is throughout properly busied with the function v, and the stream-lines are only dealt with incidentally. There is no attempt to give a physical meaning to the equation (4)." To this we have only to add that Kirchhoff proved the accuracy of his theoretical deductions by determining experimentally the form and distribution of the equipotential lines on a circular disk with two electrodes on the edge, as well as (Pogg. *Ann.* vol. lxvii. p. 344, 1846) the strength of the current at various parts of the disk; and that, from the expression for the difference of potential between the electrodes, he deduced by Ohm's law the resistance of a circular disk with two small electrodes anywhere upon it. In order to test experimentally the value thus obtained, he seems to have devised independently the arrangement now commonly known as Wheatstone's bridge; but, owing to the smallness of the resistance to be measured, he was unable to obtain satisfactory results.

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* Proc. Edinb. Roy. Soc. 1869–70, pp. 79–99.

Soon after the publication of Kirchhoff's paper, Smaasen* gave an investigation of the flow of electricity in a plane conducting sheet, in which he takes account, in determining the potential, of the electricity given off to the air, and deduces the resistance of an infinite sheet with two small circular electrodes by a process which, though longer than that employed by Kirchhoff, may be regarded as more direct. It consists in finding the resistance of the space between two lines of flow at an infinitely small distance apart, and then extending by integration the expression thus obtained so as to make it apply to the unlimited sheet. In a subsequent paper† Smaasen determined by an analogous process the resistance of a conducting sphere, or of an unlimited conductor of three dimensions‡. Smaasen's treatment of the subject is, like Kirchhoff's, based chiefly upon the mode of distribution of the potential; the only investigation we are acquainted with which deals specially with lines of flow is contained in the paper by Professor W. Robertson Smith from which we have already quoted. The starting-point adopted by Professor Smith is the same as that from which we have set out in the following communication; and, indeed, we found, after making some progress in our own work, that several of our demonstrations (which we at first thought were new) had been already given by him, while all the chief conclusions were, as we have said above, implicitly contained in Kirchhoff's original memoir. Consequently, although the present paper contains a few minor results which, so far as we know, have not been pointed out explicitly before, we do not claim for its contents any essential novelty; and our only reason for venturing to publish it is that the mode of treatment adopted in it has enabled us to establish by strictly elementary mathematical methods, and so to render accessible to a greater number of readers, many of the conclusions which previous writers had arrived at by more abstruse processes.

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* Pogg. *Ann.* vol. lxix. p. 161.

† Pogg. *Ann.* vol. lxxii. p. 435.

‡ About the same time, the same subject was taken up by Ridolfi of Florence (*Il Cimento*, An. V. 1847, May–June), whose paper, however, we only know from the references of Beetz (in Dove's *Repertorium*, vol. viii. p. 147) and Poggendorff (Pogg. *Ann.* vol. lxxii. p. 449). No reference to this paper is given either in the Royal Society's Catalogue of Scientific Papers, or in the carefully compiled "Bibliographie" of the mathematical theory of the voltaic pile in Verdet's *Conférences de Physique* (*Œuvres de Verdet*, vol. iv. p. 351).

2. In order to avoid ambiguity, it may be well to begin by giving a few general explanations and definitions of terms that will be frequently used in what follows. It will be assumed that, in all cases considered, the flow of electricity has attained a *stationary* condition—that is to say, that on the whole there is neither gain nor loss of electricity at any part of the sheet, but that at every instant the quantity that enters any part and the quantity that leaves it are equal. Positive electrodes, or such as supply positive electricity to the sheet, will be spoken of as "sources," while negative electrodes will be termed "sinks." The rate at which a source or a sink supplies or removes electricity will be spoken of as the "strength" of the source or sink.

Through every point of the sheet a straight line can be drawn such that no electricity crosses it at that point; the direction of a line so drawn through a given point is the "direction of the current" at that point. A "line of flow" or "stream-line" is a line drawn so that no electricity crosses it anywhere; or, in other words, it is a line whose direction at all points coincides with the direction of the current. The quantity of electricity flowing between two consecutive lines of flow is everywhere the same; for since no electricity crosses either of them, whatever quantity starts between them must remain between them throughout their whole course. It may be useful to observe that flow-lines are not loci of constant strength of flow, they are lines of direction simply.

The "strength of the current" at any point is given by the quantity of electricity which crosses, in unit time, a short line normal to the flow-line through the point, divided by the length of the line.

If a line be drawn so as to intersect the lines of flow everywhere at right angles, there will be no flow of electricity along it; but since the sheet on which the line is drawn is a conductor, this implies that all points of the line are at the same potential, or that the line is an "equipotential line."

3. It is obvious that a line of flow and an equipotential line might be drawn through every point of a conducting sheet traversed by a current, so that, if it were possible to draw them all, the sheet would be completely covered by two sets of lines cutting each other at right angles. The general distribution, however, of the lines of each set can be correctly shown without drawing more than a limited number, if those that are drawn are selected according to a definite and easily recognizable rule. For this purpose the most convenient rule to adopt in relation to the lines of flow, is to place them so that the total flow of electricity between each pair of consecutive lines is the same—and with regard to the equipotential lines, to place them so that the change of potential in passing from any one to the next consecutive line is the same. In the sequel, whenever a *system* of lines of flow, or of equipotential lines, is referred to, it is to be understood that the lines are placed so as to fulfil the above conditions.

One Pole in an Infinite Sheet.

4. In this case it is evident that the lines of flow are straight lines radiating out from the pole, and that the spaces between each pair of consecutive lines will convey equal currents if each line makes the same angle with the next. If a circle of radius r be drawn about the pole as centre, the quantity of electricity which crosses the whole circumference in unit of time, will evidently be the same whatever the value of r, and will be equal to the quantity Q supplied in the same time by the source. Hence, at distance r from the source, the strength of the current will be

$$\frac{Q}{2\pi r},$$

or inversely proportional to the distance r.

It is evident also, either from general considerations of symmetry, or from the condition that equipotential lines and lines of flow must intersect each other at right angles, that the equipotential curves for the case we are considering are circles having the source at their common centre.

5. If R be the resistance to the radial flow of electricity across the annular portion of the sheet bounded by circles at the potentials V_1 and V_2, we have, by Ohm's law,

$$V_1 - V_2 = QR;$$

whence, since Q is constant, equal differences of potential will be found at the boundaries of annular belts of equal resistance. The relation between the radii of successive equipotential circles with a constant difference of potential may be deduced from this condition as follows:—The resistance of any very narrow belt must be proportional to its breadth, or the difference between the radii of its external and internal circumferences; and it must be inversely proportional to the mean circumference, which, seeing that the circumference varies directly as the radius, will be simply the arithmetic mean of the external and internal circumferences. That is, if r_1, r_2, r_3 are radii of successive circles, these circles will bound spaces of equal resistance if

$$\frac{r_2-r_1}{\pi(r_2+r_1)}=\frac{r_3-r_2}{\pi(r_3+r_2)},$$

or

$$r_1r_3=r_2^2.$$

Hence the radii of successive equipotential circles form a geometrical progression whose common ratio may be called μ.

6. To find the resistance of a belt bounded by concentric circles whose radii are r_1 and r_2 respectively, let

$$\frac{r_2}{r_1}=\mu^n,$$

so that there are n equipotential circles contained in the belt, where

$$n=\frac{1}{\log\mu}\log\frac{r_2}{r_1}.$$

Denote the thickness of the conducting sheet by δ and its conductivity by κ; then, by the last section, the resistance of a narrow ring whose bounding circles have the radii 1 and $1+\Delta r(=\mu)$ will be

$$R_0=\frac{1}{\kappa\delta}\cdot\frac{\Delta r}{\frac{1}{2}(2+\Delta r)2\pi}=\frac{1}{\pi\kappa\delta}\cdot\frac{\Delta r}{2+\Delta r}.$$

But the whole belt we are considering is made up of n rings, each of the resistance R_0. So the resistance of the whole belt is

$$R=nR_0=\frac{1}{(2+\Delta r)\pi\kappa\delta}\cdot\frac{\Delta r}{\log(1+\Delta r)}\log\frac{r_2}{r_1};$$

or, letting Δr decrease without limit,

$$R=\frac{1}{2\pi\kappa\delta}\log\frac{r_2}{r_1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

7. Putting this value of R into the expression for the difference of potential of two circles of radii r_1 and r_2, we have

$$V_1 - V_2 = \frac{Q}{2\pi\kappa\delta} . \log\left(\frac{r_2}{r_1}\right),$$

whence the potential at any point of the sheet is at once given in terms of the strength of the source and the potential at unit distance from the source. Thus let V_1 be the potential at unit distance, then the potential V at any distance r becomes

$$V = V_1 - \frac{Q}{2\pi\kappa\delta} \log r. \quad . \quad . \quad . \quad . \quad . \quad (2)$$

8. The same result may be obtained without previously calculating the resistance of a belt of finite breadth as follows. The radii of successive equipotential circles, beginning at the pole itself, are (§ 5)

$$\mu^{-\infty} \ldots \mu^{-2}, \mu^{-1}, 1, \mu, \mu^2 \ldots \mu^{\infty}.$$

Let the common difference of potential between consecutive circles be Δv; then, if $r = \mu^n$, the potential at distance r is less than the potential at unit distance by n times Δv, or

$$V_r = V_1 - n\Delta v = V_1 - \frac{\Delta v}{\log \mu} \log r.$$

But $\frac{\Delta v}{\log \mu}$ is constant; and putting $1 + dr$ for μ, where dr is the breadth of the infinitely narrow belt whose inner boundary is the circle of unit radius, we have

$$\frac{\Delta v}{\log \mu} = \frac{dv}{\log(1 + dr)} = \frac{dv}{dr}.$$

Multiplying this by the circumference of the belt (2π) and by the conductivity and thickness of the sheet, we get the strength of the current across the whole belt, or

$$2\pi\kappa\delta \frac{dv}{dr} = Q,$$

$$\frac{\Delta v}{\log \mu} = \frac{Q}{2\pi\kappa\delta},$$

whence

$$V = V_1 - \frac{Q}{2\pi\kappa\delta} \log r,$$

as before.

Two equal Opposite Poles in an Infinite Sheet.

9. If there are two or more electrodes in an infinite uniform conducting sheet, the strength and direction of the current and the potential at any part of the sheet are obtained by the simple superposition of the effects which each electrode would produce at that part if it were the only electrode in the sheet. This might be regarded as probable *à priori*; and it is proved by experiment to be the case. Hence the effect of any number of poles in a sheet may be deduced by properly extending the conclusions already arrived at with regard to a single pole. We will first discuss the case of a single source and a single sink of equal strength.

10. *Lines of Flow.*—Let A (Plate IV. fig. 1) be the source, and B the sink. The flow-lines due to these, taken separately, would be two equiangular pencils of straight lines drawn outwards from A and inwards towards B. These two systems of lines subdivide the whole surface of the conducting sheet into quadrilaterals, such as P Q and Q R. The actual direction of the flow at any point P, due to the combined action of the source and sink, will evidently be intermediate between the directions of the primary streams A P and P B through the same point; similarly the direction of the resultant current at Q will be intermediate between the direction of the primary streams A Q and Q B; in other words, the line of resultant flow through P will enter the quadrilateral P Q at P, and the line of resultant flow through Q will pass at Q from the quadrilateral P Q into the quadrilateral Q R. And it can be shown that the points P, Q, and R are points on the same line of flow—that is, that a continuous curve can be drawn through P, Q, and R such that no electricity flows across it. Thus, let the points of intersection of the primary flow-line A P with the primary flow-lines through B which are nearest to P B on either side of it, be marked P' and $P_{,}$ respectively; and similarly let Q' and $Q_{,}$, R' and $R_{,}$ be the points where A Q and A R intersect the next flow-lines on either side of Q B and R B respectively. Then, since the total flow of electricity from A between the lines A P and A Q is everywhere the same and is equal to the flow towards B between the lines P B and Q B, which is likewise everywhere the same, the quantity of electricity flowing in a given time across any of the lines $P Q'$, $P_{,} Q$, $P P_{,}$, $Q' Q$, which bound the quadrilateral

PQ, is equal. Consequently, considering either of the triangles $PQ'Q$ or $PP_\prime Q$, the flow inwards across PQ' or $PP_\prime$ is equal to the flow outwards across $Q'Q$ or $P_\prime Q$; and therefore there cannot on the whole be any flow across a line drawn inside the quadrilateral from P to Q. Evidently also, by drawing a sufficient number of straight lines through A in directions intermediate between AP and AQ, and an equal number of lines through B in directions lying between PB and QB, keeping the angle between consecutive lines constant in both cases, we can subdivide the line PQ into portions as short as we please, and such that no electricity flows across any of them. Hence P and Q are points on the same line of flow; and it follows, similarly, that the point R is also situated on this line. The same kind of reasoning also proves that a second line of flow passes through the points P', Q', and R', and a third through the points $P_\prime$, $Q_\prime$, and $R_\prime$. Moreover, since the strength of the flow between the lines $P'Q'R'$ and PQR is measured by the quantity of electricity which in unit of time crosses any of the lines $P'P$, PQ', $Q'Q$, QR', or $R'R$, and the strength of the flow between the lines PQR and $P_\prime Q_\prime R_\prime$ is measured by the quantity which similarly crosses any of the lines $PP_\prime$, $P_\prime Q$, $QQ_\prime$, $Q_\prime R$, or $RR_\prime$, and since these quantities are equal to each other, it follows that the lines $P'Q'R'$, PQR, and $P_\prime Q_\prime R_\prime$ are consecutive flow-lines of a system which divides the conducting sheet into portions each of which conveys an equal current.

It is important to observe that the reasoning employed here is general, and not limited to the special case to which it has been applied. The general conclusion to which it leads may be thus stated:—If similar * systems of lines of flow be drawn, corresponding to each of two separate systems of sources and sinks, the lines of flow which would result from the combined action of the sources and sinks of both systems will be obtained by drawing curves through the alternate angles of the quadrilaterals produced by the intersections of the two primary systems of flow-lines, in directions concurrent with both the primary flow-lines that intersect each other at each angle.

11. The method which, as we have seen, allows the flow-lines

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* By *similar* systems is here to be understood systems such that the total flow between any pair of consecutive lines of the one set is the same as that between any two consecutive lines of the other set.

for two equal opposite poles to be drawn, also enables us to deduce very easily their general form. Let α be the constant angle between consecutive flow-lines of the pencil diverging from A and of that converging to B. Then, evidently,

$$\angle APB = \angle AQB = \angle ARB = \ldots = n\alpha,$$

where n is a constant integer. Also

$$\angle AP'B = \angle AQ'B = \ldots = (n+1)\alpha,$$

and

$$\angle AP_{\prime}B = \angle AQ_{\prime}B = \ldots = (n-1)\alpha.$$

Hence the lines of flow due to one source and one sink of equal strength are arcs of circles passing through the poles, each one differing from the next by a constant change $(=\alpha)$ in the angle which the radii vectores from the poles make with each other; or, what comes to the same thing, they are arcs of circles cutting each other at the poles with a constant difference of angle equal to the constant difference of angle $(=\alpha)$ between the rectilinear flow-lines which either the source or the sink would produce by itself.

12. The whole number of flow-lines is therefore $\frac{2\pi}{\alpha}$, or the same as the number of lines leaving the source or entering the sink when either of them is by itself in the sheet. This is evident also if we consider that infinitely near to either pole the effect of the other vanishes in comparison, and therefore the lines close to each pole are the same in all respects as they would be if the other pole were absent.

13. The circular form of the flow-lines for the case we are considering can be demonstrated in various other ways. We will give here one additional proof, on account of its great simplicity.

The flow through any point P due to a source at A and a sink at B, being the resultant of the currents through the same point due to A and B taken separately, will be represented in strength and direction by the third side of a triangle whose other two sides represent the currents from A and towards B, respectively, in strength and direction. But the strengths of the two component currents are inversely as the distances A P and B P respectively (§ 4): hence the following construction (fig. 2). From B draw B T parallel to A P, and make its length a third proportional to A P and P B; then P T gives the direction of the flow at P, and its length is proportional to the strength of the current at P. The similarity of the triangles A P B and P B T gives the angle B P T equal to the angle P A B; and consequently (converse of Euclid, III. 32) the locus of P is a circle through A and B.

14. It was shown in § 10 how a system of lines of flow can be traced out by successive points. To be able to draw them continuously with compasses we only require to know the position of the centres; and these are easily found from the following considerations. Since the circles of which the flow-lines are arcs pass through the poles A and B, their centres lie in the straight line at right angles to A B, through O its middle point. If C (fig. 3) be the centre of the circle which gives the flow-line through any point P, the angle at C is equal to the angle at P —the angle characteristic of the given flow-line; and therefore the angle O A C is the complement of the angle at P. Putting $AB=2a$, we have

$$OC=a\,.\tan OAC=a\,.\tan\left(\frac{\pi}{2}-APB\right).$$

Let the number of lines to be drawn be m, then the constant difference between the angles contained under consecutive lines will be

$$\frac{2\pi}{m}=\alpha,$$

and the several lines will be given by making the angle at the circumference successively equal to

$$0,\ \alpha,\ 2\alpha\ldots,\ \pi-\alpha,\ \pi,\ \pi+\alpha,\ldots 2\pi-2\alpha,\ 2\pi-\alpha,\ 2\pi,$$

or, what is the same thing, to

$$0,\ \alpha,\ 2\alpha,\ldots\pi-\alpha,\ -\pi,\ -(\pi-\alpha),\ldots -2\alpha,\ -\alpha,\ 0,$$

where the values 0, π, $-\pi$, and 2π are represented by the straight line through A and B, and negative angles indicate flow-lines passing on the lower side of A B. But, as is evident from the figure, the same circle gives the two flow-lines whose characteristic angles are $n\alpha$ and $-(\pi-n\alpha)$; hence the number of circles to be drawn or of centres to be found is equal to half the number of flow-lines; and we need only consider those characterized by angles between 0 and π, of which the complements are

$$\frac{\pi}{2},\ \frac{\pi}{2}-\alpha,\ \ldots\ \alpha,\ 0,\ -\alpha,\ \ldots\ -\left(\frac{\pi}{2}-\alpha\right),\ -\frac{\pi}{2}.$$

Consequently the required centres are obtained by drawing lines from A making the above angles with A B, and letting them intersect the perpendicular to A B through O; or, without measuring angles, by laying off from O in both directions along the perpendicular to A B lengths proportional to

$$0,\ a\tan\alpha,\ a\tan 2\alpha,\ \ldots\ a\tan\left(\frac{\pi}{2}-\alpha\right),\ a\tan\frac{\pi}{2}.$$

Plate IV. shows a system of lines of flow for which $\alpha=20°$.

15. The *strength of the current* at any point P due to a source at A and an equal sink at B is represented in the construction given in § 13 by the length of the line P T (fig. 2); but by the similarity of the triangles P B T and A P B,

$$\frac{\mathrm{PT}}{\mathrm{BT}}=\frac{\mathrm{AB}}{\mathrm{PB}};$$

or, putting $\mathrm{AB}=2a$, $\mathrm{BT}=\frac{1}{r}$, and $\mathrm{PB}=r'$, the strength of the current is

$$s=\frac{\mathrm{Q}}{2\pi}.\mathrm{PT}=\frac{\mathrm{Q}}{2\pi}\cdot\frac{2a}{rr'}. \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Hence, for given poles at a given distance apart, the strength of the current is the same at all points of the sheet for which the product of the distances from the two poles is constant; or the loci of equal flow are a system of *lemniscates**.

16. *Equipotential Lines.*—The flow-lines for the case of one source and one equal sink having been determined, the form of the equipotential lines is at once given by the consideration that the two sets of lines cut each other orthogonally (§ 2); and it is a well-known geometrical result that the system of lines orthogonal to the system of circles which, as has been seen, represents the flow-lines for this case, is another system of circles having

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* The strength of the current at various parts of a circular disk with equal opposite poles on the edge was examined experimentally by Kirchhoff, by suspending a small magnet, with a mirror attached, close above the disk, and was found to agree closely with the results of calculation. Mach has since given a more direct proof that the lines passing through points of equal flow are lemniscates. A disk of silver-leaf was coated with a thin film of wax; and on allowing a strong current to traverse it for a few moments, the wax was melted within a space bounded by a lemniscate and the edge of the disk (Carl's *Repertorium f. experim. Physik*, vol. vi. p. 11, 1870).

their centres on the line through the poles, each circle cutting this line once internally and once externally in points situated harmonically with respect to its extremities. The simplest general expression for such a system of circles is the equation

$$\frac{r}{r'}=c,$$

where c is a quantity which remains constant for each circle but varies from each circle to the next, and r and r' are the distances from any point of the curve to the source and sink respectively.

These general properties of the equipotential lines are easily established by referring to the construction employed in § 13. We there saw that P T (fig. 2) is a tangent to the line of flow through P; and consequently it is normal to the equipotential line through the same point. If we produce P T to cut A B produced in C, we have the triangles B C P and P C A similar, and

$$CP^2=CA\,.\,CB\,;$$

whence it appears that if tangents to the lines of flow be drawn from any point C in A B produced, the distance from C to the points where these tangents touch the lines of flow is constant and depends only on the distances of the point C from A and B respectively. Therefore, if a circle be drawn with centre C and radius C P, it cuts all the lines of flow at right angles, and is consequently an equipotential line. If it is only required to find the centre of the equipotential circle through any point P, the simplest method is to make an angle B P C equal to the angle B A P; then the point where P C and A B intersect is the centre to be found.

The similarity of the triangles B C P and P C A also gives

$$\frac{AP}{PB}=\frac{r}{r'}=\frac{AC}{CP};$$

or the ratio of the *radii vectores* from the two poles is constant for a given circle, as already stated.

17. The above method suffices for drawing the equipotential lines through any number of given points, but not for placing them *systematically* (or so that the difference of potential between consecutive lines may be constant). For this purpose we may have recourse to a process of superposition of the same kind as that employed (§ 10) for placing the lines of flow.

We have seen (§ 5) that the equipotential lines for a single pole are concentric circles, and that the radii of consecutive circles form a geometrical progression. To find the system of equipotential lines for two equal opposite poles, it is only needful to draw for each pole separately a system of equipotential lines with the same difference of potential between any one line and the next as it is intended that there should be between the lines of the combined system, and then to draw lines through the intersections of the two overlapping systems of circles thus obtained, taking care, in going from one intersection to the next, that the changes of potential are in opposite directions for the two primary systems taken separately. Thus, let the lines 1, 2, 3, 4 (fig. 4) represent portions of equipotential lines due to a source at A; and $1'$, $2'$, $3'$, $4'$ portions of equipotential lines due to a sink at B; and let the potential of the line 1 be V, and let that of the line $1'$ be V'; further, let the change of potential in passing from any one line to the next in either system be v, so that the potentials of the lines 2, 3, and 4 are $V-v$, $V-2v$, and $V-3v$ respectively, and the potentials of $2'$, $3'$, and $4'$ are $V'+v$, $V'+2v$, and $V'+3v$ respectively. Then, at the points where 1 and $1'$, 2 and $2'$, 3 and $3'$, 4 and $4'$ respectively intersect each other, the potentials will be the sums of the potentials of the intersecting lines; and therefore the potential at all these points is the same, namely $V+V'$. Consequently P, Q, R, and S are points on the same equipotential line. Similarly it follows that P_1, Q_1, and R_1 are points on the line whose potential is $V+V'+v$; and Q', R', and S' points on the line whose potential is $V+V'-v$. We thus get the potential of the resultant equipotential lines differing by the constant amount v, which is the same as the difference of potential of the lines of the two primary systems.

It is evident from this that any two systems of equipotential lines whatever, which have the same constant difference of potential, can be compounded, so as to give a single resultant system, by tracing lines through alternate angles of the quadrilaterals produced by the mutual intersection of the lines of the two systems, and also that the constant difference of potential between the lines of the resultant system will be the same as that between the lines of each of the component systems.

18. Let μ be the common ratio of the radii of the equipoten-

tial circles of the two primary systems considered in § 17, and let the radius of the circle 1 be μ^n and that of the circle 1′ be μ^m (see § 8); the radii of the successive circles of the one set are then μ^{n+1}, μ^{n+2}, . . ., and of those of the other μ^{m+1}, μ^{m+2}, . . . Hence for the ratio of the radii vectores from the two poles for the points P, Q, R, S, we have

$$\frac{\mu^n}{\mu^m}, \quad \frac{\mu^{n+1}}{\mu^{m+1}}, \quad \frac{\mu^{n+2}}{\mu^{m+2}}, \quad \frac{\mu^{n+3}}{\mu^{m+3}}$$

respectively; that is, we have for all these points the common ratio

$$\mu^{n-m}.$$

Similarly, we should find for the points P_1, Q_1, and R_1 the common ratio

$$\mu^{n-m+1},$$

and for the points Q′, R′, and S′ the common ratio

$$\mu^{n-m-1}.$$

Hence, not only are the equipotential lines for two equal and opposite poles characterized by a constant ratio of the radii vectores, as already proved (§ 16), but this ratio *changes in a constant ratio* on passing from any one line of the system to the next, the ratio of change $(=\mu)$ being the same as the ratio of change of radius on passing from one line of the system due to a single pole to the next.

19. The actual potential at any point of the sheet, in terms of the distances of this point from the two poles, follows directly from equation (2) in § 7. Let r be the distance of the given point from the source, and r' its distance from the sink; put V for the potential at the point due to the source alone, and V′ for that due to the sink alone. Then we have

$$V = V_1 - \frac{Q}{2\pi\kappa\delta} . \log r,$$

and

$$V' = V'_1 - \frac{Q'}{2\pi\kappa\delta} . \log r';$$

and since the source and sink are equal, $V'_1 = -V_1$ and $Q' = -Q$; therefore the resultant potential, or $V + V'$, is

$$U = \frac{Q}{2\pi\kappa\delta} . \log \frac{r'}{r}. \qquad (4)$$

This gives the potential $=0$ at all points of the straight line equidistant from the two poles, positive on the side of this line next the source, and negative on the side next the sink. Also it shows that for equal differences of potential we must have equal differences in the value of $\log \frac{r}{r'}$, which agrees with what was proved above (§ 18).

We may write (4) thus (§ 8),

$$U=\frac{Q}{2\pi\kappa\delta}\log \mu^n=n\,.\,\Delta v\,;$$

whence, regarding the source and sink for the present as mere points, the value $n=\infty$ will correspond to the former, and $n=-\infty$ to the latter, while $n=0$ denotes the straight line at an equal distance from both.

20. Let the distances from O (the middle point of A B) of the points D and D′ (fig. 2), in which the circle of potential U cuts A B, be called l and l'; then, a being as usual half the distance between the pôles,

$$\frac{a+l}{a-l}=\frac{l'+a}{l'-a}=\mu^n,$$

where μ^n is the ratio of the radii vectores of the given circle. Hence

$$ll'=a^2$$

and

$$l=a\frac{\mu^n-1}{\mu^n+1},\quad l'=a\frac{\mu^n+1}{\mu^n-1}.$$

For the radius of the circle, we have evidently

$$\rho=\tfrac{1}{2}(l'-l)=\frac{2a\mu}{\mu^{2n}-1},$$

and for the distance of its centre from O,

$$d=l+\rho=l'-\rho=a\frac{\mu^{2n}+1}{\mu^{2n}-1}\,;$$

also

$$\rho^2=\text{CA}\,.\,\text{CB}=(d+a)h=d^2-a^2,$$

where $h(=d-a)$ is the distance of the centre of the equipotential circle from the nearest pole.

21. In order to draw a system of equipotential circles with compasses, it is most convenient to have given the distances of

the centres from the nearest pole, and also the distances from the same point of one of the intersections with A B, the line joining the poles—that is, in fig. 2 the distance B C and either B D or B D′. Now

$$BC = h = d - a = \frac{2a}{\mu^{2n} - 1},$$

$$BD = a - l = \frac{2a}{\mu^{n} + 1},$$

and

$$BD = l' - a = \frac{2a}{\mu^{n} - 1}.$$

Any convenient value such as 2 or $\frac{3}{2}$ may be given to μ; or if a special number of lines be required, a value may be found to suit. Thus let L be the greatest value that is to be given to l, and m the number of lines required on each side of O. We shall have

$$\frac{a + L}{a - L} = \mu^{m}, \text{ or } \log \mu = \frac{1}{m} \log \frac{a + L}{a - L},$$

which determines the value of μ.

The successive values of B D′ are then

$$\frac{2a}{\mu - 1}, \ \frac{2a}{\mu^{2} - 1}, \ \frac{2a}{\mu^{3} - 1}, \ \ldots . \ \frac{2a}{\mu^{m} - 1}.$$

The successive values of B C are simply alternate values of these, namely

$$\frac{2a}{\mu^{2} - 1}, \ \frac{2a}{\mu^{4} - 1}, \ \ldots . \ \frac{2a}{\mu^{2m} - 1}.$$

Plate IV. shows the equipotential lines drawn for a value of $\mu = \frac{5}{3}$.

22. *Resistance.*—The resistance, in the direction of the flow, of the part of the sheet extending between two given equipotential circles follows directly from equation (4) (in § 19). Thus for the potentials of the circles characterized by the ratios $r_1 : r'_1$ and $r_2 : r'_2$ respectively, where r_1 and r_2 are distances from the source, and r'_1 and r'_2 the corresponding distances from the sink, we have

$$U_1 = \frac{Q}{2\pi\kappa\delta} \log \frac{r'_1}{r_1} \text{ and } U_2 = \frac{Q}{2\pi\kappa\delta} \log \frac{r'_2}{r_2},$$

whence

$$U_1 - U_2 = \frac{Q}{2\pi\kappa\delta} \log \frac{r'_1 r_2}{r_1 r'_2}.$$

Consequently the resistance of the part of the sheet between these circles is

$$R=\frac{U_1-U_2}{Q}=\frac{1}{2\pi\kappa\delta}\cdot\log\frac{r'_1r_2}{r_1r'_2}. \quad . \quad . \quad . \quad (5)$$

If ρ_1 is the radius of the circle which has the greater absolute potential, or the one nearest the source, and ρ_2 is the radius of the circle nearest the sink, the similarity of the triangles A P C and B P C in fig. 2, Plate IV., gives

$$\frac{\rho_1}{a+d_1}=\frac{r_1}{r'_1} \text{ and } \frac{\rho_2}{a+d_2}=\frac{r_2}{r'_2}$$

for circles round the source, and

$$\frac{\rho_1}{a+d_1}=\frac{r'_1}{r'} \text{ and } \frac{\rho_2}{a+d_2}=\frac{r'_2}{r_2}$$

for circles round the sink. Substituting these values, we get

$$R'=\frac{1}{2\pi\kappa\delta}\cdot\log\frac{\rho_2(a+d_1)}{\rho_1(a+d_2)}$$

if both circles surround the source, and

$$R'=\frac{1}{2\pi\kappa\delta}\cdot\log\frac{\rho_1(a+d_2)}{\rho_2(a+d_1)}$$

if they both surround the sink. In the former case we have $\rho_1<\rho_2$, and in the latter case $\rho_1>\rho_2$; but using ρ_1 for the radius of the smaller circle and ρ_2 for that of the larger, we may write

$$R'=\frac{1}{2\pi\kappa\delta}\cdot\log\frac{\rho_2(a+d_1)}{\rho_1(a+d_2)} \quad . \quad . \quad . \quad . \quad . \quad (6)$$

for the resistance between two equipotential circles surrounding the same pole, whether that pole be a source or a sink.

If the circles surround opposite poles, the resistance becomes

$$R''=\frac{1}{2\pi\kappa\delta}\cdot\log\frac{(a+d_1)(a+d_2)}{\rho_1\rho_2}. \quad . \quad . \quad . \quad (7)$$

In the case of circles round the same pole, if the other pole is infinitely distant, the value of R′ becomes

$$\frac{1}{2\pi\kappa\delta}\log\frac{\rho_2}{\rho_1},$$

which is identical with (1), the value found for the resistance of an annular belt of internal radius ρ_1 and external radius ρ_2; and in fact the case supposed (an infinite distance between source and sink) is physically identical with the case of a single pole in an infinite sheet.

When the circles surround opposite poles, if the radii are equal, the resistance becomes

$$\frac{1}{\pi\kappa\delta}\log\frac{a+d}{\rho} \quad \text{or} \quad \frac{1}{\pi\kappa\delta}\log\frac{a+\sqrt{a^2+\rho^2}}{\rho};$$

or when the common radius is small as compared with a,

$$\mathrm{R}=\frac{1}{\pi\kappa\delta}\log\frac{2a}{\rho}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

23. The following modified forms of (6) and (7) obtained by means of the relations (§ 20)

$$\frac{a+d}{\rho}=\frac{\rho}{h}, \quad a^2=ll', \quad 2\rho=l-l,$$

may also be noted :

$$\left.\begin{aligned} \mathrm{R}' &= \frac{1}{2\pi\kappa\delta}\log\frac{\rho_1 h_2}{\rho_2 h_1}, \\ \mathrm{R}'' &= \frac{1}{2\pi\kappa\delta}\log\frac{\rho_1\rho_2}{h_1 h_2}, \end{aligned}\right\} \quad . \quad (9)$$

$$\left.\begin{aligned} \mathrm{R}' &= \frac{1}{2\pi\kappa\delta}\log\frac{(\sqrt{l'_1}-\sqrt{l_1})(\sqrt{l'_2}+\sqrt{l_2})}{(\sqrt{l'_1}+\sqrt{l_1})(\sqrt{l'_2}-\sqrt{l_2})}, \\ \mathrm{R}'' &= \frac{1}{2\pi\kappa\delta}\log\frac{(\sqrt{l'_1}+\sqrt{l_1})(\sqrt{l'_2}+\sqrt{l_2})}{(\sqrt{l'_1}-\sqrt{l_1})(\sqrt{l'_2}-\sqrt{l_2})}. \end{aligned}\right\} \quad . \quad (10)$$

24. If two equipotential circles alone are given and it is required to find the resistance between them, the position of the poles A and B being unknown, the expressions given above for R′ and R″ are inapplicable (unless the circles are so small that their centres sensibly coincide with the poles); for in such cases the values of r, r', a, d, h, and l are all unknown. The distance, however, between the centres of the circles is directly measurable; and the resistance can be expressed in terms of this distance and of the known radii as follows.

Calling the distance between the centres D, we have for circles surrounding the same pole,

$$\mathrm{D}=d_2-d_1=\sqrt{a^2+\rho_2^2}-\sqrt{a^2+\rho_1^2},$$

which gives

$$d_1=\sqrt{a^2+\rho_1^2}=\frac{\rho_1^2-\rho_2^2-\mathrm{D}^2}{2\mathrm{D}}, \quad d_2=\sqrt{a^2+\rho_2^2}=\frac{\rho_1^2-\rho_2^2+\mathrm{D}^2}{2\mathrm{D}},$$

and

$$a=\frac{\sqrt{(\rho_1+\rho_2+\mathrm{D})(\rho_1+\rho_2-\mathrm{D})(\rho_1-\rho_2+\mathrm{D})(\rho_1-\rho_2-\mathrm{D})}}{2\mathrm{D}}.$$

Hence (6) may be written *

$$R' = \frac{1}{2\pi\kappa\delta}\log\frac{\rho_2\sqrt{(\rho_1+\rho_2+D)(\rho_1+\rho_2-D)(\rho_1-\rho_2+D)(\rho_1-\rho_2-D)}+\rho_1^2-\rho_2^2-D^2}{\rho_1\sqrt{(\rho_1+\rho_2+D)(\rho_1+\rho_2-D)(\rho_1-\rho_2+D)(\rho_1-\rho_2-D)}+\rho_1^2-\rho_2^2+D^2}. \quad (11)$$

When the equipotential circles surround different poles, we have

$$D = d_1 + d_2 = \sqrt{a^2+\rho_1^2} + \sqrt{a^2+\rho_2^2},$$

which gives

$$d_1 = \sqrt{a^2+\rho_1^2} = \frac{D^2+(\rho_1^2-\rho_2^2)}{2D}, \quad d_2 = \sqrt{a^2+\rho_2^2} = \frac{D^2-(\rho_1^2-\rho_2^2}{2D},$$

and a the same as before. Hence (7) may be written

$$R'' = \frac{1}{2\pi\kappa\delta}\log\frac{\sqrt{(\rho_1+\rho_2+D)(\rho_1+\rho_2-D)(\rho_1-\rho_2+D)(\rho_1-\rho_2-D)}-\rho_1^2-\rho_2^2+D^2}{2\rho_1\rho_2}. \quad (12)$$

25. If instead of considering the whole extent of the sheet we confine our attention to that part of it which is contained between any two lines of flow (see Plate IV.), its resistance will be given by any of the above formulæ if instead of 2π in the denominator we put the angle, say γ, which the lines of flow forming the boundaries make with each other at either pole; for the spaces between every pair of consecutive flow-lines convey equal currents; and since the difference of potential between their ends is the same for all, they all have the same resistance. Consequently the resistance of a part of the sheet made up of such spaces is inversely proportional to the number of them which compose it. Now, using as previously α to stand for the angle which two consecutive lines of flow make with each other at the poles, the total number of spaces into which the whole sheet is divided is $\frac{2\pi}{\alpha}$, and the number of spaces lying between lines of flow whose angle at the poles is γ, is $\frac{\gamma}{\alpha}$; hence, if R_γ is the resist

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* This expression was given in a slightly different form by Gaugain on the authority of Blavier (*Ann. de Chim. et de Phys.* Ser. 3. vol. lxvi. p. 203. 1862); and a demonstration has since been published by the latter (*Journal de Physique*, vol. iii. p. 115, April 1874).

ance of the part of the sheet bounded by these lines, and R the resistance of the whole of the sheet, we have

$$\frac{R_\gamma}{R} = \frac{2\pi}{\gamma}.$$

In the case of two flow-lines which form arcs of the same circle drawn through the two poles, we have $\gamma=\pi$, and therefore

$$R_\pi = 2R.$$

Hence, if a small equipotential circle is drawn round each pole, the resistance of the part of the sheet lying between these circles and contained within any circle drawn through the poles is the same, being equal to twice the resistance of the unlimited sheet between the same two equipotential circles, and also equal to the resistance of as much of the unlimited sheet as would be left after cutting out the portion bounded by any circle through the poles. Consequently if ρ is the common radius of the small equipotential circles, we have, for the resistance of a disk on the edges of which the poles are situated, and also for that of the space outside it, according to §§ 22 & 24,

$$R = \frac{2}{\pi\kappa\delta} . \log\frac{2a}{\rho} \quad \text{or} \quad R = \frac{2}{\pi\kappa\delta} . \log\frac{D}{\rho}. \qquad . \quad (13)$$

26. It may be observed that the method by which the resistance was obtained above (§ 22) is essentially the same as that adopted by Kirchhoff. The method by which we first obtained the expression, however, was founded on the consideration of the equality of the resistance of the curvilinear rectangles into which the sheet is divided by the intersecting systems of lines of flow and equipotential lines (*vide* Plate IV.). This process, which may be stated as follows, we afterwards found to be somewhat similar in principle to that employed by Smaasen.

Let two circular electrodes be placed upon the sheet so as to coincide with two of the natural equipotential circles, which for simplicity we will assume to have the same radius ($=\rho$). The spaces between consecutive lines of flow are of equal resistance, and may for the present purpose be regarded as so many independent conductors connecting the two poles and combined in multiple arc. Consequently the resistance of the sheet which is made up of these spaces, is equal to the resistance of one of them divided by their whole number. These spaces in their turn are each composed of $2n$ equi-resisting rectangles arranged in series; so the resistance of each space is $2n$ times the resistance of one of the rectangles, $2n$ being (as is evident from Plate IV.) the whole number of equipotential lines in the sheet which are not obliterated by the electrodes.

Now consider the resistance of one of the four rectangles at the middle of the figure. Its length $OD = l = a\frac{\mu-1}{\mu+1}$ (§20) (for, $n=0$ giving the straight equipotential line through O (§ 19), the next one will be given by $n=1$). Its breadth b, which in fig. 3 is denoted by OQ, will be

$$a\cot\frac{\pi-\alpha}{2} = a\tan\frac{\alpha}{2}.$$

Calling δ the thickness and κ the conductivity of the sheet as before, the resistance of the rectangle will be approximately

$$R_0 = \frac{l}{b\kappa\delta} = \frac{1}{\kappa\delta\tan\frac{1}{2}\alpha}\cdot\frac{\mu-1}{\mu+1}.$$

The number of equipotential lines between the electrodes is $2n$ if the electrodes are of equal size and coincide with the nth equipotential circle on each side of the middle line. The radius of this circle is given in § 20 as

$$\rho = \frac{2a\mu^n}{\mu^{2n}-1};$$

solving which we get

$$\mu^n = \frac{a \pm \sqrt{a^2+\rho^2}}{\rho}.$$

The positive sign only is admissible, since μ is essentially positive; whence the number of lines

$$2n = \frac{2}{\log\mu}\log\frac{a+\sqrt{a^2+\rho^2}}{\rho}.$$

So, then, the resistance of a strip between two consecutive flow-lines is

$$R_1 = 2nR_0 = \frac{2}{(\mu+1)\kappa\delta\tan\frac{1}{2}\alpha}\cdot\frac{\mu-1}{\log\mu}\log\frac{a+\sqrt{a^2+\rho^2}}{\rho}.$$

The number of flow-lines in the sheet is (§ 12)

$$\frac{2\pi}{\alpha};$$

therefore the resistance of the whole sheet is approximately

$$R=\frac{\alpha}{2\pi}R_1=\frac{1}{(\mu+1)\pi\kappa\delta}\cdot\frac{\alpha}{\tan\frac{1}{2}\alpha}\cdot\frac{\mu-1}{\log\mu}\log\frac{a+\sqrt{a^2+\rho^2}}{\rho}.$$

Now, make the rectangles infinitely small (and therefore rectilinear), by letting α approach 0 and μ approach 1; then

$$R=\frac{1}{\pi\kappa\delta}\log\frac{a+\sqrt{a^2+\rho^2}}{\rho}.$$

the same expression as that obtained at the end of § 22 for the same case.

The resistance of the segment contained between two flow-lines intersecting each other with the angle γ is, under the same circumstances (§ 25),

$$R_\gamma=\frac{2}{\gamma\kappa\delta}\log\frac{a+\sqrt{a^2+\rho^2}}{\rho}. \quad . \quad . \quad . \quad . \quad . \quad (14)$$

Any number of Poles in an Infinite Sheet.

27. *Lines of Flow.*—The equation to the flow-lines for two equal and opposite poles, A and B, in an unlimited sheet, was obtained in § 11 in the following form,

$$\angle APB=n\alpha,$$

where n is a parameter whose value changes by unity on passing from one line to the next, and α is the angle between adjacent rays of the equiangular pencil of flow-lines produced by either pole separately. If the straight line AB be taken as the axis of x, the above equation may be written

$$\angle APB=\angle PBX-\angle PAX=n\alpha.$$

Moreover it was shown, in § 10, that the system of flow-lines resulting from the composition of any two similar systems is obtained by drawing lines through the alternate angles of the quadrilaterals produced by the mutual intersection of the lines of the component systems, in directions concurrent with both the flow-lines which intersect each other at each angle. Hence, if the poles A and B (Pl. IV. fig. 1) were of the same sign (both sources or both sinks), the lines of flow of the resultant system, instead of passing through the points P, Q, R and P_1, Q_1, R_1, &c., as there represented, would pass through the supplementary angles of the quadrilaterals, namely from Q' to P_1, from R' to Q_1, &c. It is evident that, as we pass from point to point of a line so drawn, the angles which the *radii vectores* from the poles make with the axis of x change equally, but in opposite directions; hence this equation may be written

$$\angle \mathrm{PBX} + \angle \mathrm{PAX} = n\alpha.$$

28. If, instead of the line through the poles, any other straight line in the plane of flow be taken as the direction of reference for measuring the angles made by the *radii vectores*, the form in which the equations to the lines of flow have just been given will still be applicable. Thus, if the line X (Pl. V. fig. 1), making any angle δ with A B, be taken as the direction of reference, and if a and b be the points where it is met by P A and P B respectively, it is clear that the angles P a X and P b X are each of them less than the previous angles P A X and P B X by the angle δ, but that for each flow-line they give a constant sum or difference according as the poles are of the same or of opposite kinds. Hence, denoting the angles which the *radii vectores* make with any fixed line by θ_1 and θ_2 respectively, and agreeing to distinguish sources and sinks by a difference of intrinsic sign of the angles corresponding to them, we may write the equation to the flow-lines due to two equal poles, whether similar or opposite, in the general form

$$\theta_1 + \theta_2 = n\alpha.$$

29. In like manner, the lines of flow due to any number of equal poles are represented by the equation $\Sigma\theta = n\alpha$, the angles which the *radii vectores* from the several poles make with any fixed line being respectively denoted by θ_1, θ_2, ... In order to prove this, it is sufficient to show that, if the expression holds good for any given number of poles, it will still be true for one pole more; for since the formula has been proved for two poles, this would enable us to advance, by successive additions of one pole at a time, to any number.

Let, then, the curves $\mathrm{P}''\mathrm{Q}'\mathrm{R}$, $\mathrm{P}'\mathrm{Q}\mathrm{R}_1$, and $\mathrm{P}\mathrm{Q}_1\mathrm{R}_2$ (Pl. V. fig. 2) represent parts of three successive lines of flow due to any number k of equal poles, and let them be characterized respectively by the following values of $\Sigma\theta$, namely $(n+1)\alpha$, $n\alpha$, and $(n-1)\alpha$. If there is another pole of the same strength at L, which we will suppose in the first instance to be a *source*, the pencil of flow-lines diverging from it with the common difference of angle α will intersect the flow-lines from the k sources, producing a number of quadrilaterals through whose opposite angles the lines of flow will pass which result from the action of the pole at L combined with that of all the rest (§ 10). If the direction of the flow along the lines of the component systems be as represented by the arrow-heads* in the figure, one line of the resultant system will pass through the points P′ and Q′, another through P, Q, and R, and a third through Q_1 and R_1. Let the lines joining the source L with the points P, Q, and R cut the fixed line X in *p*, *q*, and *r* respectively; then, for the flow-line through Q due to the *k* poles, we have by hypothesis

$$\theta_1+\theta_2+\ldots+\theta_k=n\alpha;$$

and for the line through the same point due to the source L we have

$$\angle \mathrm{Q}q\mathrm{X}=m\alpha,$$

and therefore, for both together,

$$\Sigma\theta+\mathrm{Q}q\mathrm{X}=(n+m)\alpha.$$

For the point P, we have similarly

$$\Sigma\theta+\mathrm{P}p\mathrm{X}=(n-1)\alpha+(m+1)\alpha;$$

and for the point R we have

$$\Sigma\theta+\mathrm{R}r\mathrm{X}=(n+1)\alpha+(m-1)\alpha.$$

Consequently for each of these points, which we already know are situated on the same flow-line of the resultant system, we have

$$\theta_1+\theta_2+\ldots+\theta_k+\theta_l=(n+m)\alpha.$$

In the same way, we should get for the points P′ and Q′ situated on the line of flow of the resultant system which lies next to the line through P, Q, and R on the left, the respective values which follow:

$$\Sigma\theta+\mathrm{P}'p\mathrm{X}=n\alpha+(m+1)\alpha,$$
$$\Sigma\theta+\mathrm{Q}'q\mathrm{X}=(n+1)\alpha+m\alpha,$$

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* For the purpose of the argument in the text, we may assume arbitrarily either the *direction of flow* along the assumed system of lines of flow, or the *direction of increase* in the value of *n* (in *nα*), but not both; for a definite assumption regarding one of these conditions determines the other also. If the system of flow-lines for any given set of poles is built up by successive repetitions for one pole at a time of the construction given in § 10, it will be found that the conventional distinction between positive and negative angles leads to the following rule as to the connexion between the two above-named conditions:—Going along a given flow-line *with the flow*, any line on the *left hand* corresponds to a *higher* value of *n* than the given line, while a line on the *right hand* corresponds to a *lower* value of *n*.

and therefore for each of them

$$\theta_1+\ldots+\theta_k+\theta_l=(n+m+1)\alpha.$$

The same reasoning gives for the points Q_1 and R_1 on the line of flow next on the right to that through P, Q, and R,

$$\theta_1+\ldots+\theta_k+\theta_l=(n+m-1)\alpha.$$

30. Next suppose the additional pole at L to be a *sink*; then, since the resultant flow must be everywhere concurrent with both the components, the new flow-lines will pass through the other pair of angles of each quadrilateral; for instance, three consecutive lines will pass through the points P' and Q_1, through P'', Q, and R_2, and through Q' and R_1; and by applying the same considerations as above, it will be seen that these lines are characterized by the following values of the expression

$$\theta_1+\ldots+\theta_k+\theta_l,$$

namely:—for the line through P' and Q_1,

$$(n-\overline{m+1})\alpha;$$

for that through P'', Q, and R_2,

$$(n-m)\alpha;$$

and for that through Q' and R_1,

$$(n-\overline{m-1})\alpha.$$

Hence it follows that sources and sinks affect the value of the sum of angles denoted by $\Sigma\theta$ in opposite ways, and therefore that, as assumed provisionally above (§ 28), sources and sinks must be distinguished by a difference in the intrinsic sign of the angles which the *radii vectores* drawn from them make with the fixed line. When this is done, the equation

$$\Sigma\theta=n\alpha$$

is true for the lines of flow produced by any number of equal poles, whether of the same sign or not.

Further, if we have poles of unequal strengths supplying (or removing) quantities of electricity in the unit of time denoted, say, by Q_1, Q_2, . . ., these may be regarded as each of them produced by the coalescence of a corresponding number of equal poles supplying in unit of time a quantity of electricity q which is taken small enough to be a common measure of all the quantities Q_1, Q_2, ... In applying the general formula to such a case, the value of θ corresponding to each pole would have to be taken a number of times equal to the number of constituent poles of strength q required to make up the actual pole. Hence

$$\Sigma(Q\theta)=Q_1\theta_1+Q_2\theta_2+\ldots=n\alpha \quad . \quad . \quad . \quad (15)$$

is a perfectly general expression for the lines of flow produced by any number of poles of any strength.

31. *Equipotential Lines.*—It was shown in § 17 that the system of equipotential lines resulting from the composition of any two systems, for which the constant difference of potential on passing from one line to another is the same, is given by drawing lines through the alternate angles of the quadrilaterals formed by the mutual intersection of the lines of the component systems, if the angles are taken in such order that on going from any one to the next we pass, in one of the component systems, to a line of higher, and in the other to one of lower potential. We know also (see § 5) that the equipotential lines due to a single pole are concentric circles whose radii vary according to the terms of a geometrical progression, the potential increasing in the case of a source as the radii decrease, and, in the case of a sink, as the radii increase. Hence it follows very simply, by the application of reasoning exactly analogous to that employed in § 29, that if P be a point on an equipotential line due to any number of equal sources at the points A, B, C, ..., and sinks at the points A′, B′, ...,

$$\frac{AP \,.\, BP \,.\, CP \ldots .}{A'P \,.\, B'P \ldots .}=\mu^n=\text{constant},$$

where μ is the constant ratio of the radii of consecutive equipotential circles due to a single pole, while n is a number characteristic of the particular line on which the point P is situated and increasing by unity if this point passes from any given line to the line of next lower potential. For shortness we may put r_1 for AP, r_2 for BP, ..., and write the above equation thus,

$$\frac{r_1 \,.\, r_2 \,.\, r_3 \ldots .}{r'_1 \,.\, r'_2 \ldots .}=\mu^n,$$

or

$$\Sigma \log r=n\log\mu.$$

32. No part of the above reasoning will be disturbed if we suppose that each of the equal poles hitherto discussed emits (or absorbs) in unit of time a quantity of electricity equal to unity; and as this supposition will simplify the consideration of poles

of unequal strength, we will adopt it. Then, when the poles are unequal, let them severally emit in unit of time quantities of electricity represented respectively by Q_1, Q_2, Q_3, ... (the absorption of electricity by sinks being reckoned as negative emission), the effect will be the same as if Q_1, Q_2, ... poles of unit strength coincided at the points occupied by the actual poles. Hence, in such a case, the *radii vectores* from the several poles must be taken Q_1, Q_2, ... times in the above formula, and it thus becomes

$$\left.\begin{array}{l} r_1^{Q_1}.\, r_2^{Q_2}.\, r_3^{Q_3} \ldots\ldots = \mu^n \\ \text{or} \qquad \Sigma(Q \log r) = n \log \mu, \end{array}\right\} \quad \ldots\ldots \quad (16)$$

which is a quite general expression for the equipotential lines due to any distribution of poles of any strength.

33. To find the actual potential at any point due to a given set of poles, we must recur to equation (2), § 7. Let V_1, V_2, ... be the potentials which the various poles would produce at the given point if each acted separately, and let ϕ_1, ϕ_2, ... be the potentials produced by each separately at unit distance from itself; then, taking the other symbols in the senses already defined, we have

$$V_1 = \phi_1 - \frac{Q_1}{2\pi\kappa\delta} . \log r_1,$$

$$V_2 = \phi_2 - \frac{Q_2}{2\pi\kappa\delta} . \log r_2;$$

$$\vdots \qquad \vdots$$

and for the resultant potential at the given point,

$$U = \Sigma V = \Sigma\phi - \frac{1}{2\pi\kappa\delta} . \Sigma(Q \log r). \quad \ldots \quad (17)$$

Since, for a given set of poles, $\Sigma\phi$ is constant, this gives for a locus of constant potential

$$\Sigma(Q \log r) = \text{const.} = 2\pi\kappa\delta(\Sigma\phi - U),$$

which is in accordance with (16); for we may write

$$\Sigma\phi - U = n\Delta v,$$

and by § 8, when $Q=1$, we have $\log \mu = 2\pi\kappa\delta . \Delta v$.

34. As already observed (§ 1), both the general equations

$$\Sigma(Q\theta) = \text{const.} \quad \text{and} \quad \Sigma(Q \log r) = \text{const.}$$

were given thirty years ago by Kirchhoff, who obtained the former as a purely mathematical consequence of the latter. In fact, as the systems of lines represented by them are orthogonal to each other, it follows that their first derived functions differ only in one having $\frac{dy}{dx}$ where the other has $-\frac{dx}{dy}$; hence differentiation and subsequent integration, after making this substitution, convert one expression into the other*, This relation between the two equations is of importance, since the form of the equipotential lines can be readily determined experimentally, whereas no practicable method exists for ascertaining experimentally the course of the lines of flow.

35. Before going further we may point out some general properties of lines of flow and equipotential lines. Physically considered, a line drawn from any one pole to any other, so that no electricity crosses it at any point, may be regarded as an independent line of flow, as was done in treating the case of two equal and opposite poles. The lines, however, given by the equation $\Sigma(Q\theta) = \text{const.}$ are continuous curves each of which passes through every pole; and, in a mathematical sense, the whole of the curve given by any one value of the constant must be regarded as a single line of flow. When flow-lines are spoken of in the sequel, it is to be understood that they are *complete* in the above sense. If all the poles are of equal strength, every flow-line passes once through each of them; hence, if they are unequal, every line passes through each of them respectively as

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* Let the coordinates of the several poles be $(a_1 b_1)$, $(a_2 b_2)$, &c. The distances of a point from each will be $r_1 = \sqrt{(x-a_1)^2 + (y-b_1)^2}$, $r_2 =$ &c., and the angles which the joining lines make with the axis of x will be given by $\tan\theta_1 = \frac{y-b_1}{x-a_1}$, $\tan\theta_2 =$ &c. Then

$$\frac{d}{dx}\Sigma(Q\log r) = \Sigma\,.\,Q\frac{d}{dx}\log\sqrt{(x-a)^2+(y-b)^2}$$

$$= \Sigma\,.\,Q\left\{\frac{x-a}{(x-a)^2+(y-b)^2} + \frac{y-b}{(x-a)^2+(y-b)^2}\cdot\frac{dy}{dx}\right\}.$$

Now make the substitution, equivalent to writing $\phi + \frac{\pi}{2}$ for ϕ, and integrate,

$$\int \Sigma\,.\,Q\left\{\frac{y-b}{(x-a)^2+(y-b)^2} - \frac{x-a}{(x-a)^2+(y-b)^2}\cdot\frac{dy}{dx}\right\}dx$$

$$= \Sigma\,.\,Q\tan_{-1}\frac{y-b}{x-a} = \Sigma(Q\theta).$$

often as it contains the greatest common measure of the strengths of the whole set. Moreover, if on any flow-line there is a point of no flow, the line will intersect itself at this point one or more times. Separate flow-lines cannot intersect each other anywhere except at a pole.

Equipotential lines form closed curves which always surround one or more poles. At points where the strength of the current is nothing, the same equipotential line cuts itself once or oftener; in such cases there is at least one pole within each loop formed by the line. Separate equipotential lines never intersect each other.

The method of superposition, by which any system of flow-lines or equipotential lines can be drawn, is easily carried out by drawing, in rather strong lines on white paper, one of any two systems that are to be compounded to a resultant system, and drawing the other component system on tracing-paper. Placing the two drawings one over the other in any required manner, and laying a second piece of tracing-paper on the top, the curves which pass through the intersections of the two component systems can be drawn at once. The resultant system may then, if required, be recompounded with one of the components by the same process.

36. The general course of the lines of flow and equipotential lines for a few of the simpler cases is shown in Plate V.

Two equal similar poles (fig. 3).—The equation to the lines of flow for this case is

$$\theta_1+\theta_2=n\alpha,$$

which is equivalent to

$$x^2-y^2-2xy\cot n\alpha=a^2,$$

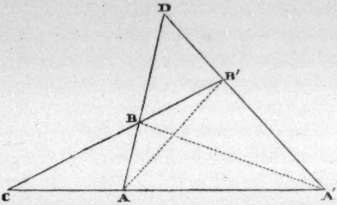
if the straight line through the poles is taken as the axis of x, and its middle point as the origin. They are a system of rectangular hyperbolæ cutting each other at the poles with a constant difference of angle ($=\alpha$). The value $n\alpha=\pi$ gives the two axes, the origin where they intersect being a point of no flow. The equipotential lines for the same case are a system of lemniscates, $r_1r_2=\mu^n$, and are identical with the loci of equal flow for two equal opposite poles (§ 15). The strength of the current at any point at a distance r from the origin is $s=\frac{Q}{\pi}\cdot\frac{r}{r_1r_2}$; and consequently the loci of constant flow are the same as the equipotential lines for three equal poles—two of the same sign, and one of the opposite sign halfway between them, as shown in fig. 4. The strength of the current on the self-cutting curve is $\frac{Q}{2\pi a}$. For all other values of s the curve consists of two separate branches; when s is $> \frac{Q}{2\pi a}$, one branch encloses each pole; and when s is $< \frac{Q}{2\pi a}$, one branch surrounds both poles and the other surrounds neither, being within the central loop of the self-cutting curve. Each of the loci cuts any straight line through the origin harmonically with respect to its intersections with the circle $r=a$.

Three equal poles.—A special case coming under this head, which admits of some important applications, is that in which there are three equal poles in a straight line, one of the outside ones being of opposite sign to the other two—for instance, two sources A and A′, and one sink C. In this case the flow-line given by $\theta_1+\theta_2-\theta_3=\pi$ consists of two branches, one of which is the straight line passing through the poles; while the other is a circle whose centre is the sink C (fig. 5), and whose radius C P is a mean proportional between the distances from the sink to the two sources respectively (or $\mathrm{CP}=\sqrt{\mathrm{CA}\,.\,\mathrm{CA}'}$). If the two sources coincide, the systems of lines of flow and equipotential lines become what are shown in fig. 6, and the circular branch of the flow-line $n\alpha=\pi$ passes through the pole of double strength. In accordance with what was said above (§ 34), every flow-line of the system passes twice through the double source and once through the sink.

Four equal poles.—Four equal poles of the same sign situated at the corners of a square give the system of flow-lines represented in fig. 7. The two diagonals of the square and the two straight lines through the middle point parallel to the sides of the square are given equally by the values $n\alpha=\pi$ and $n\alpha=0$. For $n\alpha=\frac{1}{2}\pi$ we get a curve of four branches, one of which is situated symmetrically in each quadrant.

A special case of four equal poles, which is important in consequence of its being susceptible of experimental verification, is presented by a combination of two sources and two sinks arranged in a manner that may be regarded as a duplication, with inversion of signs, of the system of three poles shown in fig. 5. Let the pole at C be a sink, those at A and A′ being sources. Let two additional sinks be placed at B and B′ (fig. 8), points on another straight line through C, and let a source equal to them be put at C; then the three poles of this new system would give by themselves, as one of their flow-lines, the straight line C B and the circle with centre C and radius $= \sqrt{\text{CB} . \text{CB}'}$. If, however, the three new poles be made equal to the three poles of the first set, the sink originally at C and the equal source now put there will exactly compensate each other, and the combined system will be reduced to two sources at A and A′ and two sinks equal to them at B and B′. If, further, the points B and B be taken so that $\text{CB} . \text{CB}' = \text{CA} . \text{CA}' = \text{CP}^2$, the circle of radius CP will be a common line of flow in each of the systems due to C, A, and A′ and to C, B, and B′, and will therefore remain a line of flow in the resultant system (see fig. 9) due to A, A′, B, and B′. Moreover, since these four points are placed so that one circle can be drawn through them all, it is easy to see that this circle will be another flow-line of the resultant system; for the four poles may be grouped in two ways into two pairs of equal opposite poles (either A, B and A′, B′, or A, B′ and A′, B), each of which would separately give this circle as a line of flow. If the straight lines AB and A′B′ are drawn and produced to their intersection at D (fig. 8), a circle drawn about this point as centre with radius $= \sqrt{\text{DB} . \text{DA}} = \sqrt{\text{DB}' . \text{DA}'}$ will be an equipotential line common to the systems due respectively to the source and sink at A and B, and to those at A′ and B′; consequently this circle is also an equipotential line of the resultant system due to all four poles. Another equipotential line of the resultant system would

Fig. 8.



be the circle (in this case imaginary) drawn with the point of intersection of AB′ and A′B as centre so as to cut each of these straight lines in points harmonically situated with respect to their extremities.

Again, if four equal poles be taken situated as in the last example, but so that poles of the same sign are diagonally opposite each other—in other words, if the signs of either A and B or of A′ and B′ be interchanged, the circle passing through the four poles will still be a flow-line of the resultant system, but C, as well as D, will now be the centre of an equipotential circle, while the imaginary circle will be a line of flow.

37. *Resistance.*—Equation (17) gives, for the difference of potential of any two points whose respective distances from the several poles are $r_1, r_2, r_3, \ldots$, and $r'_1, r'_2, \ldots$ the value

$$\begin{aligned} U - U' &= \frac{1}{2\pi\kappa\delta}\left[\Sigma(Q \log r') - \Sigma(Q \log r)\right] \\ &= \frac{1}{2\pi\kappa\delta}\Sigma\left(Q \log \frac{r'}{r}\right), \end{aligned}$$

since, for each pole, there are the corresponding terms $Q_n \log r_n$ and $Q_n \log r'_n$. If there are altogether K poles, k being of one sign and k' of the opposite sign, where k is not less than k', and if, further, they are all of the same strength, the quantity of electricity crossing each complete equipotential line in unit of time is kQ; consequently the resistance of the portion of the sheet lying between the equipotential lines which pass through the given points is

$$R = \frac{U - U'}{kQ} = \frac{1}{2\pi\kappa\delta \cdot k} \cdot \Sigma\left(\log \frac{r'}{r}\right). \quad . \quad . \quad (18)$$

This formula is in principle quite general; but the practical application of it in actual cases requires that we should know the position of the poles from which the distances $r_1, r_2, \ldots$ are to be measured; and these cannot be ascertained (or at least not by elementary methods), except for comparatively few and simple cases; for although it is comparatively easy to determine the equipotential lines for a given set of poles, the inverse problem, of finding the distribution of poles required to produce equipotential lines coinciding with two given curves on the conducting sheet, presents in general very great mathematical difficulties, and has hitherto received only partial solutions.

38. We will give here, in the first place, the approximate application of the general formula to the case of two sources and two equal sinks at the angles of a quadrilateral inscribed in a circle and so placed that unlike poles are diagonally opposite each other. This is the arrangement shown in figs. 8 and 9, where A and A′ may be taken as sources and B and B′ as sinks. It is evident that the equipotential lines very near the poles will each consist of two branches, one of them surrounding one source (or sink), and the other surrounding the other source (or sink), and also that they will be very approximately circles having the poles at their centres. Hence, if the sources and sinks are formed by four circular electrodes, whose common radius ρ is a small fraction of the distance between any two of them, and if they are so placed that the distances of their centres A, A′, B, and B′ from a fixed point C fulfil the condition $CA \,.\, CA' = CB \,.\, CB'$ (which is equivalent to saying that their centres are on the circumference of one and the same circle), we may without serious error regard the circles of contact between the electrodes and the conducting sheet as forming together a pair of equipotential lines due to such a distribution of poles as that referred to at the beginning of this paragraph. Then, taking points on the circumference of the circles round A and B as the points to which the values of r and r' in equation (18) respectively refer, we have approximately

$$\Sigma\left(\log\frac{r'}{r}\right) = \log\frac{(AB)^2 \,.\, AB' \,.\, A'B}{\rho^2 \,.\, AA' \,.\, BB'} ;$$

and since for the case supposed $k = k' = 2$, the resistance of the part of the sheet extending between the pair of circles round A and A′ and the pair round B and B′ is represented with similar approximation by

$$R = \frac{1}{4\pi\kappa\delta} \,.\, \log\frac{(AB)^2 \,.\, AB' \,.\, A'B}{\rho^2 \,.\, AA' \,.\, BB'} \quad . \quad . \quad . \quad . \quad (19)$$

If a circle be drawn with the centre C and radius

$$CP = \sqrt{CA \,.\, CA'} = \sqrt{CB \,.\, CB'},$$

this circle will coincide, as already pointed out (§ 35), with a flow-line of the system due to the combined action of the four poles. Consequently no electricity passes into or out of this circle; and therefore the whole of the electricity supplied by the source A flows to the sink B *inside* the circle (fig. 9), while the whole of what is supplied by the source A′ flows to B′ *outside* the circle. It follows, since the sources A and A were assumed of the same strength, that the resistance offered by the part of the sheet lying within the circle of radius CP to the flow of electricity between the electrodes A and B is the same as the resistance of the part of the sheet lying outside this circle to the flow of electricity between the electrodes A′ and B′. Hence also the resistance of a disk bounded by the circle in question and containing the two electrodes A and B is equal to *twice* the resistance of the entire sheet to all four poles, and is therefore represented by

$$R' = \frac{1}{2\pi\kappa\delta} \cdot \log \frac{(AB)^2 \, . \, AB' \, . \, A'B}{\rho^2 \, . \, AA' \, . \, BB'}, \quad . \quad . \quad . \quad (20)$$

which is the formula referred to in § 1 (page 115) as having been given by Kirchhoff for the resistance of a circular disk with two small circular electrodes anywhere upon it.

39. As already stated, this formula is only approximate, and in certain special cases it entirely fails. For instance, if one (or both) of the poles passes to the edge of the disk, then, in order that the circumference may still continue to be a line of flow, the second pole of the same sign must coincide with it; consequently in such a case AA′ or BB′, or both, will vanish, and the expression for the resistance fails by becoming infinite. The reason evidently is that equation (20) was got by assuming ρ to be very small in comparison with any of the distances between the poles; and this can no longer be true when any two of them coincide. By slightly modifying the notation, we can obtain an expression which does not fail in the same way. Thus, let the points to which the values of r_1, r_2, . . . refer be respectively a point F where the straight line A B cuts the equipotential circle round A, and a point G where the same line cuts the equipotential circle round B. Then we have AF = BG = ρ, and the expression for the resistance of the sheet to the current from all four poles becomes

$$R = \frac{1}{4\pi\kappa\delta} \cdot \log \left(\frac{BF \, . \, B'F}{AF \, . \, A'F} \cdot \frac{AG \, . \, A'G}{BG \, . \, B'G} \right), \quad . \quad (19a)$$

while the resistance of the disk bounded by the circular flow-line may be written

$$R' = \frac{1}{2\pi\kappa\delta} \cdot \log \frac{AG \, . \, BF \, . \, A'G \, . \, B'F}{\rho^2 \, . \, A'F \, . \, B'G}. \quad . \quad . \quad (20a)$$

In this form the expression for R′ admits of extension to some special cases. For example, let the pole A pass to the edge of the disk, while B goes to the centre; then A′ will coincide with A, and B′ will go to an infinite distance. We have then sensibly AG=BF=A′G= the radius of the disk (=P), also A′F=ρ, and the resistance to the flow between the pole at the edge and the pole at the centre of the disk becomes

$$\frac{3}{2\pi\kappa\delta}\,.\log\frac{P}{\rho}\,.\quad .\ .\ .\ .\ .\ .\ .\ . \qquad (21)$$

If both poles go to the edge, A′ coincides with A, and B′ with B, the system being reduced to two equal opposite poles at a distance AG=BF=$2a$ from each other. In accordance with this, the resistance of the whole sheet becomes

$$\frac{1}{\pi\kappa\delta}\,.\log\frac{2a}{\rho},$$

which is the value already given for the same case in equation (8).

40. If, all else remaining as before, the signs of the poles A′ and B′ are interchanged (that is, if A′ becomes a sink and B′ a source), the circle drawn with the centre C and radius $\sqrt{\mathrm{CA}\,.\,\mathrm{CA}'}$ becomes an equipotential line instead of a line of flow, the circle with centre D and radius $\sqrt{\mathrm{DB}\,.\,\mathrm{DA}}$ remains an equipotential line, and the only circular lines of flow are the circle through the four poles and the imaginary circle (§ 36) whose centre is the intersection of AB′ and A′B. The resistance of the sheet to the flow of electricity between this new arrangement of poles is given by interchanging in (19a) the signs of the *radii vectores* drawn from A′ and B′, whereby we get

$$R=\frac{1}{4\pi\kappa\delta}\,.\log\frac{\mathrm{AG}\,.\,\mathrm{BF}\,.\,\mathrm{A'F}\,.\,\mathrm{B'G}}{\rho^2\,.\,\mathrm{A'G}\,.\,\mathrm{B'F}}. \qquad (19b)$$

The equipotential circles with the centres C and D being common to the system due to the source at A and sink at A′, and also to that due to the sink at B and source at B′, and having equal but opposite potentials when taken as belonging to either of these systems separately, will, in the system due to the four poles, form the two branches of the line of zero potential. The other equipotential lines of the system due to the four poles consist also each of them of two branches, both of which never lie within the same one of the two circles in question. From this it follows that each circle divides the sheet into two equi-resisting portions; and consequently the resistance of each of them is

$$R''=\frac{1}{2\pi\kappa\delta}\cdot\log\frac{AG\,.\,BF\,.\,A'F\,.\,B'G}{\rho^2\,.\,A'G\,.\,B'F}[*]. \quad . \quad . \quad (22$$

41. Since half the lines of flow due to each pole lie within the circle drawn through them all, the resistance of the portion of the sheet bounded by this circle is (§ 25) twice the resistance of the unlimited sheet; consequently it is equal to R′ or to R″ according to the arrangement of the poles.

42. It was pointed out in § 38 that the resistance of the part of the sheet outside the circular flow-line with centre C, to the flow from the source A to the sink B, is the same as the resistance of the part inside this circle to the flow from A to B. Accordingly the value of equation (20) remains unaltered when A′ and B′ are put for A and B, and *vice versâ*, and at the same time ρ^2 (or $\rho_A\rho_B$) is replaced by $\rho_{A'}\rho_{B'}=\rho_A\rho_B\left(\frac{A'B'}{AB}\right)^2$. The circles round the two sources then coincide approximately with the two branches of a single equipotential line; and the same is true for those surrounding the sinks.

Similar remarks are applicable to equation (22).

Experimental verifications of some of the conclusions here arrived at will be given in Part II. of this communication.

XVI. *The Electrolysis of certain Metallic Chlorides.*

By J. H. GLADSTONE, *Ph.D., F.R.S., Fullerian Professor of Chemistry in the Royal Institution, and* ALFRED TRIBE, *F.C.S., Lecturer on Chemistry in Dulwich College.*

WE have previously shown that nitrate of copper brought into tension by silver and copper in conjunction is decomposed by free oxygen in solution[†]. Thinking that chlorine

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[*] It may be noted that by adding together the values of R′ and R″ we get $\frac{1}{2\pi\kappa\delta}\cdot\log\left(\frac{AG\,.\,BF}{\rho^2}\right)^2$, which, written in the simpler form

$$\frac{2}{\pi\kappa\delta}\cdot\log\frac{AB}{\rho},$$

is the resistance of a circular disk on whose edge the poles A and B are placed (see equation 13).

[†] Proc. Roy. Soc. vol. xx. p. 290.

might be substituted for oxygen, we commenced some experiments, employing chloride of copper, and observed some facts which seemed to have an interest from their bearing on the causation of galvanic action.

It is known that if metallic copper be placed in a solution of cupric chloride, it will slowly become covered with a crystalline deposit of the insoluble cuprous chloride:—

$$Cu + CuCl_2 = 2CuCl.$$

We found that when metallic copper and platinum are connected by a wire and immersed in cupric chloride, the insoluble salt forms not only upon the copper, but also on the platinum plate, as a white crystalline body. This deposit may generally be observed in about two minutes when the plates are three quarters of an inch apart. The formation of cuprous chloride upon the platinum plate takes place about equally rapidly in solutions containing 2·5 or 10 per cent. of salt. With a 20 per cent. solution the deposit was smaller, and with 40 per cent. practically *nil*, although there was abundant formation of cuprous chloride upon the *copper* plate.

We satisfied ourselves that the action took place equally well in solutions from which oxygen had been rigidly excluded, and also that a current passed from the copper to the platinum through the liquid—that is, from the metal of higher to that of lower potential.

In order to test whether this electrolysis of cupric chloride into CuCl and Cl could be effected by weak currents *ab extra*, we tried the effect of a zinc-platinum cell excited by common water and with platinum electrodes, and found that cuprous chloride deposited upon the negative electrode and chlorine at the positive, a little of which entered into combination with the platinum, but the greater part passed into the liquid. A cell excited with dilute sulphuric acid acted in a similar manner. A single Grove's cell gave for the first two or three minutes cuprous chloride on the negative platinum electrode, but afterwards metallic copper, while chlorine always formed at the positive plate.

As zinc immersed in a salt of copper is capable of throwing down that metal, an experiment was tried with plates of zinc and platinum in connexion immersed in the chloride; the result was a more energetic action than with a copper-platinum couple similarly arranged, and besides a thick coat of cuprous chloride the edges of the platinum were incrusted with metallic copper. A similar magnesium-platinum couple gave a similar result, but with a decidedly greater proportion of metallic copper.

As there are two chlorides of mercury, similar to the two chlorides of copper, analogous experiments were tried with solution of corrosive sublimate.

A small bell-jar which terminated in a corked orifice was inverted and partly filled with mercury, over which was poured a solution of mercuric chloride; a wire passed from the mercury through the cork to a plate of platinum which hung in the solution, without, however, touching the mercury. On pouring the mercuric chloride upon the mercury, the metallic surface was at once dimmed by a film of mercurous chloride, which increased in quantity; and in the course of an hour or two the insoluble chloride appeared also on the platinum plate, and in twenty-four hours it was sufficiently thick to permit of its removal. On repeating this experiment with gold instead of platinum, the same mercurous chloride was deposited; but at the same time the gold plate was amalgamated, showing that the reduction of the mercuric chloride had not stopped at the first stage, but had actually proceeded to the separation of the metal itself. A similar gold plate immersed in the same solution of corrosive sublimate, but not in connexion with the mercury, showed no trace of deposit or amalgamation; and it was ascertained that gold alone has no power of decomposing moistened mercurous chloride.

An experiment was made with a current *ab extra*. A cell of Grove's was found to decompose mercuric chloride with the formation of the mercurous compound at the negative platinum electrode, while chlorine was given off at the positive one. This is in unison with what was found in the case of the copper salts.

That this action does not depend on the insolubility of the *-ous* chlorides was proved by the behaviour of the iron salts. Ordinary metallic iron is capable of reducing the ferric to the ferrous salt at the common temperature; thus:—

$$2\,FeCl_3 + Fe = 3\,FeCl_2.$$

Platinum does not effect such a reduction; but when the iron is connected with platinum the change takes place more rapidly, and the reduced salt forms also on the negative metal. This, of course, does not render itself evident by any deposit, nor by any appreciable change of colour at first; but if the platinum plate be lifted out of the solution, and the liquid clinging to it be allowed to drain on to a paper moistened with some ferridcyanide of potassium, Turnbull's blue is the result. Or if a few drops of the ferridcyanide be mixed with the solution of the ferric chloride, on the junction of the iron and platinum the blue colour makes its appearance against each metal. Of course care was taken that the original salt contained no acid. The solution employed was one of 3·5 per cent. in strength.

If plates of magnesium and platinum be immersed in ferric chloride, metallic iron quickly makes its appearance on the platinum plate.

With a weak external battery and platinum electrodes the salt was resolved into chlorine and ferrous chloride, but with a strong battery into chlorine and iron—thus affording another instance of the close analogy between the primary electrolysis produced in the cell itself, and the secondary electrolysis that may be produced by an external battery.

May 22, 1875.

XVII. *A new Polariscope.* *By* Professor W. G. Adams.

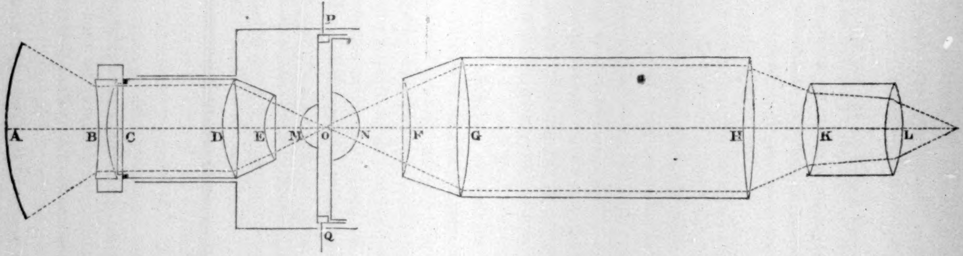
In devising this instrument, the principal objects in view have been:—

(1) To obtain an extensive field of view.

(2) To afford a means of measuring the rings and the angles between the optic axes of biaxal crystals.

(3) To have a means of immersing the crystal in a liquid in those cases in which the optic axes are too far apart to be seen in air.

These advantages have been obtained by modifying the positions and focal lengths of the lenses usually employed in table polariscopes, so that the rings of a crystal are best seen when there is a space of 1¼ inch between the two lenses, one on either side of the crystal. Into this space is introduced a central piece, consisting of a circular box with deep plano-convex lenses fixed, one in the bottom and the other in the top of the box, in such a position that their curved surfaces have a common centre of cur-



A. Concave mirror.
B. Double-concave lens 1 inch in diameter,
C. Tourmaline or other polarizer.
D. Double-convex crossed lens 1 inch in diameter, $1\frac{1}{4}$ inch focal length.
E. Convexo-plane lens $\frac{4}{5}$ inch in diameter, 1 inch focal length.
F. Plano-convex lens 1 inch in diameter, $1\frac{1}{4}$ inch focal length.
G. Double-convex crossed lens $1\frac{1}{2}$ inch in diameter, $1\frac{3}{4}$ inch focal length.
H. Double-convex lens $1\frac{1}{2}$ inch in diameter, 3 inches focal length.
K L. Nicol's prism and eyepiece.
M and N. Portions of hemispherical lenses with O as the common centre of curvature of their spherical surfaces. Radius of curvature of M is $\frac{1}{4}$ inch, and of N is $\frac{3}{8}$ of an inch, and the distance between the lenses M and N about $\frac{1}{8}$ of an inch
P and Q. The ends of the axis supporting the box fixed so that the axis passes through the point O.
The dotted lines show the path of the light through the instrument.

vature, with their flat faces turned towards one another and enclosing the crystal between them. The box can turn about an axis passing through the common centre of curvature.

The form and position of Mirror and Lenses for the Polariscope.

A is a concave mirror about $1\frac{1}{2}$ inch in diameter, such as is ordinarily employed for illuminating microscopes, and mounted in the same manner. A double-concave lens, B, 1 inch in diameter, is placed so as to have its focus at the same point as the principal focus of the mirror. The rays coming from the mirror will then be parallel after passing through the lens. To diminish aberration as much as possible, the radius of curvature of the first face of the lens should be about six times the radius of its second face. The parallel rays then fall on a tourmaline or other polarizer, C, the diameter of which should be nearly equal to that of the lens. They then fall on a double-convex crossed lens D, the first face of the lens having the greatest curvature, so as to diminish the aberration as much as possible. This lens is 1 inch in diameter and $1\frac{1}{4}$ inch focal length.

At a distance of $\frac{1}{4}$ of an inch from this lens is placed a convexo-plane lens E, $\frac{4}{5}$ of an inch in diameter and 1 inch focal length.

These three lenses, B, D, and E, may be fixed in the same piece; and with the above arrangement, rays which are parallel before falling on the mirror will be brought to a focus at a distance of half an inch on the outside of the last lens E. The rays then fall on the central piece M N, consisting of two plano-convex lenses, which are nearly hemispheres, enclosing the crystal between them. The first of these lenses, M, is 5 millims. in thickness, and the radius of its spherical surface is 6·4 millims., or $\frac{1}{4}$ of an inch; the other lens, N, is 7 millims. in thickness, and the radius of its spherical surface is 9·6 millims., or $\frac{3}{8}$ of an inch; and they should be so placed that the two centres of curvature are accurately at the same point O. The distance between them will then be 4 millims., which is very nearly the thickness of most specimens of crystals. If lenses with the same curvatures but of thicknesses $4\frac{1}{2}$ and $6\frac{1}{2}$ millims. respectively be used in place of M and N, there will be a breadth of 5 millims. between them when they are in the best position. This central piece should be so placed that O, the centre of curvature of the two curved surfaces of the lenses, coincides with the focus to which the rays converge after they come out of the first system of lenses. The rays will then pass through this central piece without having their direction altered, since the crystal will refract the light nearly to the same extent as glass, and the light will pass nearly as if it passed through the centre of a sphere of glass.

After passing through this central piece, the rays diverge from the common centre of curvature and fall on a plano-convex lens, F, 1 inch in diameter, and placed about $\frac{5}{8}$ of an inch from the common centre of curvature. Its focal length should be $1\frac{1}{4}$ inch. At a distance of half an inch from this lens is another lens, G, $1\frac{1}{2}$ inch in diameter and $1\frac{3}{4}$ inch focus, so as to make the rays again parallel. Then at a distance of 3 inches is placed a lens, H, whose focal length is rather less than 3 inches and whose diameter is $1\frac{1}{2}$ inch; and above this the Nicol's prism, K L, with any other apparatus, such as the beautiful arrangements of Mr. Spottiswoode for showing the effects of quartz and other crystals on polarized light. At the principal focus of the last lens, H, should be placed crossed spider-lines; and for accurate measurement a simple lens or eyepiece should be added above the Nicol's prism.

The object of receiving the light from the mirror on a double-concave lens is to avoid aberration and the consequent loss of light; but this involves the use of a large piece of tourmaline. The difficulty is got over in existing polariscopes by employing two convex lenses, causing the rays to cross the axis very near the polarizing tourmaline, and then making them parallel by means of a second convex lens. The central piece may be added to any polariscope provided there is sufficient room between the two systems of lenses to admit the crystal and two lenses, M, N, and to allow of motion about the axis through the common centre of curvature of the surfaces of these two lenses. The two parts, B C D E and F G H, should be so arranged that when the crystal alone is placed at O the rings are best seen; then the crystal should be placed in the box between the two lenses M and N in the bottom and in the cover of the box. This box should be supported on two wires, P and Q, forming an axis passing through O, and should be large enough to contain the cork in which the crystal is placed. The axis is supported in a brass tube, which forms one piece with another brass tube which fits on the piece B C D E. A semicircular disk of brass with its arc graduated is fixed on the large brass tube about the axis P as a centre; and on this axis is an index for reading-off the angle through which the axis is turned. On turning the box about the axis, no change is produced in the rays which pass through the centre O of the curved surfaces of the lenses; but when a crystal is introduced, the rays may be sent through it in any desired direction on turning the axis, so that either of the optic axes of a biaxal crystal may be brought into the centre of the field. Thus the angle between the optic axes may be measured. The central piece M N is made in the form of a box, so that a liquid may be introduced into it for measurements in those cases in which the optic axes are too far apart to be seen in air.

The advantages to be obtained by the use of this central piece are:—

(1) The extension of the field of view. If the angle in air corresponding to the field of view is 74° without the central piece, then the angle will be increased to about 128° when the central piece is introduced, the central piece giving the same angle in glass that is given without it in air. The field of view may be made to include both optic axes of topaz of Brazil.

(2) When the plane containing the optic axes is at right angles to the axis P Q, either of the optic axes of a biaxal crystal or any ring may be brought into the centre of the field of view where the spider-lines cross one another, and the angles between them accurately measured. Instead of employing a lens for an eyepiece, the centre of the field may be determined by fixing crossed spider-lines at a point halfway between the lenses D and E. These lines will be brought to a focus by the system of lenses E, M, N, F, and G at a distance to the right of G equal to about one tenth of an inch; and at this focus another set of crossed spider-lines should be placed, so that the two sides may by their coincidence determine the centre of the field of view. The spider-lines should be in the focus of the lens H, so that they may be seen on looking through the Nicol's prism.

XVIII. *An Experiment for showing the Electric Conductivity of various forms of Carbon.* *By* H. Bauerman, *F.G.S.*

The following simple method of exhibiting the conducting-power of carbon was brought to my notice by my friend Mr. W. J. Ward, of the Metallurgical Laboratory of the Royal School of Mines, as having been shown to him several years since by Dr. von Kobell, of Munich. As I have not found any account of it published, I have ventured to bring it before this Society.

A fragment of the substance to be tested, whether charcoal, coke, anthracite, or other form of carbon, is held between the jaws of a pair of tongs formed by bending a strip of sheet zinc into a horseshoe form, and immersed in a solution of cupric sulphate. If the carbon is a non-conductor, the copper salt is decomposed, and deposit of copper only takes place on the immersed surface of the zinc; but when it possesses a high degree of conductivity a zinc-carbon couple is formed, and deposit of copper takes place on the surface of the carbon as in ordinary electrotyping.

Of the different forms of carbon experimented upon, the most rapid results have been obtained with some American anthracites, and coals that have been subjected to the action of intruded igneous rocks. The most remarkable of these is an anthracite from Peru, which contains a large amount of sulphur in organic combination, and is found in a nearly vertical position, interstratified in quartzite, in the high plateau of the Andes, about 13,000 feet above the sea-level, near Truxillo. It is probably of secondary age, the metamorphism having taken place at the time of the great trachytic outbursts which form the gold- and silver-bearing rocks of the adjacent mining-district. This is coppered by immersion almost as readily as graphite. The anthracite of Pennsylvania possesses the same property, but not in quite such a high degree. The Heathen coal of South Staffordshire, when altered by the intrusion of the "white-rock" trap, is more slowly coppered; but this is probably due to the resistance interposed by the numerous laminæ of calcite filling the fractures in the mass of the coal, which renders the conductivity less perfect. A specimen of coal from Bengal, altered in the same manner by intrusion of igneous rock, behaves much in the same way as coke, being coppered directly. This is rather remarkable, as this coal is a very impure one, and contains such a large quantity of water very intimately combined, probably as a hydrated silicate interspersed through the mass, as to decrepitate explosively when suddenly heated.

The ordinary Welsh anthracite does not appear to be a conductor by this method; but after having been heated to a full red heat it conducts electricity freely. The lowest temperature at which this change takes place appears to be somewhere between the melting-points of zinc (430° C.) and silver (1000° C.), as fragments of anthracite packed in a thin clay crucible and plunged into molten zinc were not found to be altered, but were changed when heated in a bath of melted silver. These limits, although considerably wide apart, are interesting as giving a possible clue to the temperature at which anthracitic metamorphism of coals has been effected in different districts. Mr. W. C. Roberts has recently shown that the alloys of silver and copper have very definite melting-points; it will be possible therefore to determine more nearly the lowest temperature necessary to produce the change.

In the South Wales anthracite district it is well known that no great amount of disturbance has taken place in the position of the coal-seams, while in North America and Peru the change has been accompanied with much more violent action, as evidenced by the greater disturbance of the rocks; and probably a correspondingly higher degree of heat was developed in the mass. The evidence afforded by the coals that have been actually altered by intruded rocks, and must have been highly heated, appears to bear out this view. On the other hand, long-continued exposure to a lower temperature might possibly produce the same effect, and further experiments upon this point would be desirable.

XIX. *On an Apparatus to illustrate the Formation of Volcanic Cones.* *By* C. J. Woodward, *B.Sc.*

A short time ago I spent some weeks in Central France, and had many opportunities of studying the character of the volcanic cones which are so abundant in that district. The many sections of these cones that may be seen at Graveneire and other places, where the scoriæ is obtained for industrial purposes, show their structure clearly, and one can realize the truth and simplicity of the theory accounting for their formation. The theory is so obviously correct that it has not, so far as I know, been subjected to experiment on a small scale. The object of this paper is to describe an apparatus that may be used in the lecture-room to build up model cones and craters.

Soon after my return I tried an experiment at some Iron-works to make such a cone. A large sheet of iron with a hole in the centre was placed over a pipe proceeding from the blast. A quantity of cinders and ashes were supplied to the pipe, and in a short time a cone was built up. For the purpose of lecture-illustration the apparatus about to be described is convenient, and shows very clearly the formation of volcanic cones. It consists of a wooden trough about 18 inches long, with sloping sides; at the bottom of the trough there is a bladed screw to carry forward the ashes, sawdust, or other material used, to an opening through which air from a powerful bellows is forced upward in a vertical stream. A board 3 or 4 feet square with a hole in the centre is placed over the air-jet, so that on turning the screw the sawdust is carried into the stream of air; it is then thrown vertically upward through the hole in the upper board, and on falling down builds up a cone having many of the peculiarities of a natural one. Thus, the angle of slope is almost identical; the sorting of materials is similar, the heaviest portions being near the crater, while the finer particles and dust are carried to a great distance. If, while the miniature eruption is going on, the oxy-hydrogen light be directed horizontally on to the cone, it will be seen that the dust is carried to a distance of many feet. Using alternately sawdust of different colours, the structure of the cone is well seen on making a section of it. This is easily done by cutting vertically through the cone with a sheet of cardboard and sweeping off one half of the mound into a tray. If the jet of air be vertical and the air of the room is still, the lip of the crater is horizontal; but if the apparatus be near the door or where there is a draught, the materials are carried to one side, and the cone is built up much higher on one side than the other. The same thing, of course, will occur if the jet is not vertical; and as from accidental circumstances the jet frequently changes its direction, a considerable variety of cones is sure to be obtained in a series of experiments.

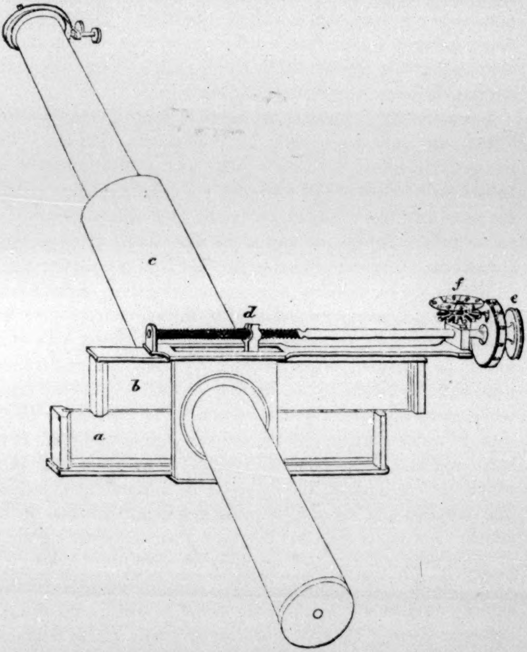
Birmingham, May 15, 1875.

June 26, 1875.

XX. *On a New Form of Micrometer for use in Spectroscopic Analysis.* *By* W. M. WATTS, *D.Sc., Physical-Science Master in the Giggleswick Grammar School.*

IN determining the position of lines in a spectrum by the use of a micrometer-eyepiece or divided arc, it is often difficult to see the cross-wires distinctly without admitting extraneous light, which with faint spectra frequently cannot be done. I have sought to overcome this difficulty by substituting some one known line of the spectrum itself for the cross-wires, and thus to measure the position of unknown lines, by bringing this index-line successively into coincidence with them. Thus, for example, the sodium-line (which is present in nearly every spectrum whether it is wanted or not) may be made to move along under the spectrum, and the displacement necessary to make it coincide with the lines to be measured may be determined by the readings of a micrometer-screw. To accomplish this a convex lens of about 2 feet focus is placed in front of the prism of the spectroscope, between the prism and the observing-telescope, and is divided along a line at right angles to the refracting edge of the prism. One half of the lens is fixed; the other half is made to slide over it by means of a micrometer-screw. When the movable half of the lens is in its normal position, the only effect is to alter the focus of the telescope slightly; but when it is made to slide over the fixed half, the refraction of the prism is increased or diminished, half of the spectrum appears to move over the other half, and the sodium-line, or any other convenient line of reference, can be brought into coincidence with the lines to be measured.

The figure represents the micrometer as applied to a direct-vision spectroscope of Browning. *a* and *b* are the two strips



cut from a lens of about 2 feet focus, placed in front of the compound prism contained in the body *c*. Of these *a* is fixed; but *b* can be made to slide over *a* by means of a screw *d*; and the displacement is measured by means of the graduated head *e* and the counter *f*.

In order to reduce the indications of this instrument to wave-lengths, I have constructed a series of interpolation-curves from the data obtained by careful observation of the solar spectrum. One of these curves is for use when the sodium-line ($D_1=5889$) is employed as the reference-line, another when the Fraunhofer line *b* ($b_1=5183$) is used, and so on. For use with terrestrial spectra of bright lines, the lithium-, sodium-, and thallium-lines, the hydrogen-lines obtained from a vacuum-tube, or for faint spectra the carbon-lines obtained from a Bunsen burner, furnish convenient reference-lines in sufficient number. The curves are drawn on such a scale that a difference in wave-length of 100 tenth-metres* is represented by four inches, and one turn of the micrometer-screw is represented by one inch.

It requires 21·91 turns of the screw to bring the sodium-line (5892) into coincidence with the thallium-line (5349). The interpolation-curves are very regular and of slight curvature. In fact the readings of this micrometer may be treated in exactly the same way as the readings obtained by means of a graduated arc or bifilar eyepiece, or any of the other forms of measuring-apparatus.

Within the limits allowed by the construction of the instrument, the displacements measured by the micrometer-screw necessary to bring a given reference-line into coincidence with other lines to be measured are proportional to the angles between these lines and the reference-line as they would be obtained by a telescope moving over a graduated arc. The extreme displacement of which the upper half of the lens is capable is 0·69 of an inch; and assuming a refractive index of 1·5 for the glass of which the lens is made, the radius of its surface is 23 inches. The deviation of a ray of light passing through the lens in its extreme position is thus that due to a prism of refracting angle $2\sin^{-1}\frac{0{\cdot}69}{23{\cdot}00}$; that is, about twice the angle whose sine is 0·03. If, now, we calculate the minimum deviation which a ray of light suffers in passing through the lens at points whose distances from the axis of the lens are represented by 0·01, 0·02, and 0·03 respectively, we obtain angles very nearly in the ratio of these numbers.

The following numbers show the deviation suffered by a ray of light in passing through a glass prism ($\mu=1{\cdot}5$) of varying angle:—

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* A tenth-metre is $\left(\frac{1}{10}\right)^{10}$ of a metre, or the ten-millionth of a millimetre.

Angle of prism.	Deviation.		
	°	′	″
$2\sin^{-1}0{\cdot}01$	0	34	22
$2\sin^{-1}0{\cdot}02$	1	8	46
$2\sin^{-1}0{\cdot}03$	1	43	12

The angles 0° 34′ 23″, 1° 8′ 46″, 1° 43′ 9″ would be as the numbers 1, 2, 3.

I add the results of a few measurements made with the new micrometer, designed to test the accuracy of which it is capable. Twenty different readings of the point at which there is coincidence of the lenses gave the following results:—

8·36 8·29 8·35 8·32 8·36 8·34 8·38 8·37 8·34 8·37
Mean 8·35

8·37 8·41 8·26 8·28 8·31 8·35 8·21 8·35 8·38 8·35
Mean 8·33

To bring the sodium-line into coincidence with the thallium-line the following displacements were necessary:—

21·90 21·90 21·89 21·90 Mean 21·90
21·92 21·93 21·94 21·88 Mean 21·92

I have generally taken the mean of four readings as giving the true position of a line.

The light emitted by burning magnesium examined with the spectroscope shows, besides the bright lines corresponding to *b*, a series of bands nearer to F. I had measured the wave-lengths of these bands with the greatest accuracy I could attain by the use of Browning's 6-prism automatic spectroscope in conjunction with a bifilar micrometer-eyepiece. The reference-lines employed were the bright lines of the spark due to zinc, cadmium, iron, and air, and certain Fraunhofer lines. The following results were obtained for the *first* and *second* bands:—

First band. 5006·5 from Fraunhofer lines.
5009·0 from zinc-line 4924
5008·0 from air-lines $\left\{\begin{matrix}5002\\5005\end{matrix}\right.$
5006·5 from Fraunhofer lines.
5007·7 from air-lines $\left\{\begin{matrix}5002\\5005\end{matrix}\right.$

Second band. 4995·0 from Fraunhofer lines.

4997·0 from air-lines $\left\{\begin{matrix}5002\\5005\end{matrix}\right.$

4997·4 from air-lines $\left\{\begin{matrix}5002\\5005\end{matrix}\right.$

The numbers finally adopted are given below for the whole nine bands; and with them are compared the results recently obtained with the new micrometer. The reference-line employed for these last measurements was the bright line corresponding to *b* present in the same spectrum.

Wave-lengths of Lines of Magnesium Oxide.

Automatic spectroscope with micrometer-eyepiece.	Direct-vision spectroscope with new micrometer.
5007	5006·0
4997	4996·0
4986	4985·5
4975	4974·5
4963	4963·5
4948	4948·5
4934	4934
*4918?	4924
	4914

Each of these results is the mean of five readings. A re-measurement of the first two bands by displacement of the lines in the opposite direction gave 5006·2 and 4996·1.

To test the capabilities of the new micrometer on faint spectra, two determinations were made of the wave-length of a line in the oxide-of-carbon spectrum given by a vacuum-tube enclosing coal-gas. The wave-length of this line I found before by measurement with a three-prism instrument to be 5195. It was obtained very faintly by a feeble discharge through the tube, and was measured by using as reference-line the lines given by a Bunsen flame.

Two successive determinations (each the mean of four readings) gave 5196 and 5196.

The advantages of the new form of micrometer seem to be (1) great precision in results, (2) convenience in use. My thanks are due to Mr. Browning for the skill with which he carried out my wishes in the construction of the instrument.

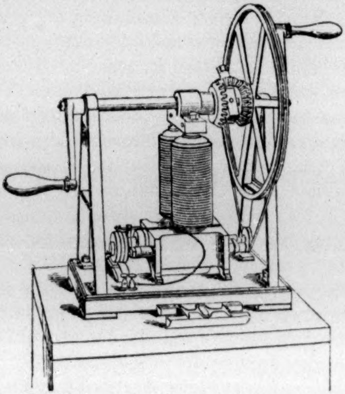
XXI. *On a new Form of Dynamo-Magneto-Electric Machine.*

By S. C. Tisley.

In the first machines constructed by Siemens and Wheatstone in 1867 (see Royal Society's Transactions) the power of augmenting the magnetism by successive currents developed from the original residual magnetism contained in the iron, was fully demonstrated, and it was shown that the power of the machine could thereby be increased to a great extent; but the only means of obtaining external work was by the insertion in the circuit of a magnet or coil so that the secondary discharge could be utilized. Sir Charles Wheatstone also showed that a great part of the current could be shunted through a platinum wire, care being taken that the resistance of the platinum wire was sufficient to compel a large portion of the current to pass round the electromagnet. In the same year the writer designed a machine in which two separate armatures were employed, one for magnetizing the machine, the other for external work. From this machine very good results were obtained.

The machine now about to be described is a still further modification, in which the greatest amount of simplicity and effective power are combined.

The apparatus consists essentially of an electromagnet with shoes, forming a groove in which a Siemens armature is made to revolve: this is much the same as the original machines made by Siemens and Wheatstone; but there is a difference in the break or commutator, from which the currents are conducted by means of two springs or rubbers. It consists of three rings; one of these rings is complete for three quarters of the circle, the other quarter being cut away; another ring is cut away three quarters, leaving the one quarter. Between these two rings is a third ring insulated and connected with the insulated end of the wire wound round the armature; on this centre ring are projecting pieces, one a quarter of a circle and the other three quarters, so arranged as to complete the two outer circles. The rubber spring which comes in contact with the quarter of the middle ring is connected with the electromagnet of the machine, and the armature is so arranged that at the time of contact the best magnetizing current is developed. The other spring rubber is in connexion with the wire on the armature during the other three quarters of its revolution; and this is connected with any external piece of apparatus to be worked.



By this arrangement, the alternate currents being utilized, they are all in the same direction; and owing to the length of contact the whole of the current is obtained in the best condition for heating wires, decomposing water, giving an electric light, &c.

At present only a model machine has been constructed on this principle, the armature, on which is wound about 50 feet of cotton-covered copper wire, No. 16, B. W. G., measureing 5 inches in length and 2 inches in external diameter. The magnet is formed with about 300 feet of covered copper wire, No. 14, B. W. G.; and the whole instrument, without the driving-gear, weighs 26 lbs.; with this apparatus 8 inches of platinum wire, .005, can be made red-hot, water can be rapidly decomposed, &c.

The armature is specially constructed so as to prevent the accumulation of heat, a fault to which every class of dynamo-magneto-electric machine is liable. It is made in two halves, a groove of a zigzag form being cast in each half, so that when the two are screwed together a continuous channel is maintained through the bearings for a current of cold water to pass during the whole time the machine is at work.

The advantages secured by these arrangements are extreme simplicity, and a small number of parts, only one armature and one wire being used.

The principle of utilizing the alternate currents is also applicable to machines constructed with multiple armatures; and the economy thereby resulting would prove of great advantage, as the power of the machine could be varied by throwing into the electromagnets either every other current, or every fourth, sixth, or eighth current, according to the strength required in the machine, the whole of the other currents being utilized for the electric light or otherwise.

XXII. *On Stationary Liquid Waves.*

By FREDERICK GUTHRIE.

THE following numerical results of experiments may be of use in the further study of wave-motion. They refer to stationary waves mainly, and include those of circular and those of rectangular troughs. The guiding idea was:—to deduce the velocity of wave-progression under different conditions from the frequency of recurrence of a given phase in the same place, in a manner similar to that employed in the measurement of the velocity of a sound-wave through a solid by the pitch of the note when a certain length of the solid vibrates longitudinally; and, by making use of reflection, to bring into a compact form for study the effects of wave-progression.

Circular Troughs.

§ 1. The first condition of motion examined in circular troughs is that which I shall call binodal. This motion can be easily set up in a cylindrical trough of water if it be not much less than 9 inches in diameter, nor the water much less than 6 inches deep. The bottom of an empty beaker glass serves well to generate the waves, on account of its buoyancy. This is placed on the centre of the water, and moved down and up at a rate dependent upon the diameter of the vessel. The motion of the water soon guides the hand, and, being judiciously humoured establishes itself and continues for many minutes with great regularity. The number of times in one or more minutes that the crest appears in the middle may be easily counted if the reflection of a window-sash is seen obliquely on the liquid surface.

§ 2. *Influence of amplitude.*—Before examining the influence of diameter (which with stationary waves amounts to an examination of the influence of wave-length), a brief examination of the influence of Depth, Temperature, Density, and Amplitude had to be undertaken, to determine at all events the margin of variation admissible without vitiation of the result. The amplitude was measured by standing a brass wire graduated in millimetres, and fastened to a leaden foot, in various parts of the trough after the wave-system was established.

§ 3. A cylindrical vessel, B, of tin plate had the mean diameter 456 millims. (It was slightly elliptical, 455, 457 millims.) The depth of the water in it was 290 millims., and its temperature 16° C. Being set in binodal motion, the gauge was put in and the amplitude allowed to subside to 30 millims. at the centre.

In the 1st minute 122 recurrences were counted.
„ 2nd „ 122·5 „ „
„ 3rd „ 122 „ „

There was an interval of $10''$ between the 1st and 2nd minute, and an interval of $15''$ between the 2nd and 3rd minute. The amplitude of motion at the centre had now sunk to less than 1 millim. This shows that in this system amplitude or wave-height is without sensible influence. This is confirmed by the results of the following experiment, in which the initial amplitude at the centre was again 30 millims., and in which the number ot pulsations made in $4'$ was counted in four experiments.

(1) Amplitude at first at centre 30 millims.; in $4'$ there were 490
(2) „ „ „ „ 489·5
(3) „ „ „ „ 489·5
(4) „ „ „ „ 489
or B_n mean in $1' = 122{\cdot}4$.

§ 4. *Effect of Temperature.*—The water in vessel B was now heated to 41° C., or through 25° C. It had the same depth as before. The following three observations were made:—

(1) In $3'$ there were 367 pulsations.
(2) „ „ 366 „
(3) In $3'\,25''$ „ 420 „

which gives for 1′ the mean number 122·4, precisely the same result as for T=16°. Temperature, therefore, has no appreciable influence, and its variation may be neglected.

§ 5. *Effect of Depth.*—For the examination of this influence the vessel B was again employed. The maximum amplitude at the centre was in all cases, at the beginning, about 30 millims.

Depth.	Time.	Number of pulsations counted.	*n*.
millim.			
290	4	489·5	122·4
270	4	490	122·5
250	3	368	122·6
230	4	490	122·5
210	3	368	122·6
190	3	367	122·3
170	3	367	122·3
150	3	366	122·0
130	3	364·5	121·5
110	3	360	120·0
90	2	234	117·0
70	2	225	112·5
50	1	102	102·0

At a depth of 30 millims. no fundamental undulations could be preserved. In order to see whether the effect of shallowness took most effect on waves of larger or smaller amplitude, the depth 70 millims. was restored and an initial central amplitude of 30 millims. was given. In the first minute 113 were counted; after a lapse of 10″ the same number 113 were counted in the second minute. A second experiment, in which the interval between the two minutes was 15″, gave for both minutes exactly the same number 113. The general conclusions to be drawn from these results appear to be that the normal rate of pulsation is not reached unless there be at least a depth of about 130 to 170 millims. (say 6 inches), and that the effect of shallowness is as great on small as on larger amplitudes. No doubt the diminution in rate is due to the increased drag upon the bottom of the vessel. It is at about this depth that the to-and-fro motion of the particles at the bottom of the vessel ceases to be sensible.

But the examination of the effect of depth is not complete unless we know whether that effect is a function of the diameter or wave-length. Accordingly a circular vessel D was taken of the diameter 314 millims., with the following result:—

Depth of water.	Number of pulsations in 1 minute.	Mean n.
millim.		
150	149, 149, 149	149
130	149, 149, 148	148·7
120	by interpolation	147·9
110	147, 147, 147	147
100	by interpolation	146·9
90	145, 145, 145	145·7
80	143, 143, 143	143
70	142, 142, 142	142
60	139, 139, 139	139
50	134, 134, 134	134
40	126, 125, 126	125·7

It appears, then, that in the vessel D, whose diameter is 314 millims., the effect of the shallowness begins to be felt at rather lesser depths than is the case in the wider troughs. At depths of 30 millims. and under, the fundamental system cannot maintain itself.

§ 6. *Effect of variation in chemical nature.*—A cylindrical vessel, which I shall call D, had the diameter of 321 millims. It was filled to a depth of 80 millims. with methylated spirit. This gave in each of three experiments 143 pulsations per minute. Water of the same depth gave also exactly 143. We may conclude that, as in other oscillatory motions, the density of the substance is without effect. This was shown experimentally, in the case of waves progressing in rectangular troughs, fifty years ago by the brothers Weber.

§ 7. *Effect of variation in the diameter.*—I used chiefly four cylindrical vessels. Their dimensions were as follows:—

Vessel A, mean diameter at surface of water 595 millims.
„ B, „ „ „ 456 „
„ C, „ „ „ 366 „
„ E, „ „ „ 300 „

Experiments with A (depth = 290 millims.).

In $3'$ there were 321 pulsations,
„ $5'$ „ 535 „
„ $4'$ „ 427 „
„ $10'$ „ 1071 „
or mean $A_n = 106·96$.

Experiments with B were given in § 3, whence it appeared that (depth =290 millims.)

$$\text{mean } B_n = 122{\cdot}4.$$

Experiments with C (depth = 285 millims.).

In 3′ there were 411 pulsations,
„ 3′ „ 410 „
„ 3′ „ 410 „
or mean $C_n = 136$.

Experiments with E (depth = 870 millims.).

In 1′ there were 151 pulsations.
„ 1′ „ 151 „
„ 1′ „ 151 „
or mean $E_n = 151$.

Putting the results together,

	Diameter.	Number of pulsations per 1′.
A . . .	595	106·96
B . . .	456	122·4
C . . .	366	136·73
E . . .	300	151

§ 8. In the binodal system of undulations here set up it is clear that the wave-length is equal to the diameter of the cylinder. If, then, the rate of progression is directly proportional to the square root of its length, we must have $v = C\sqrt{\text{diameter}}$, where C is a constant. But the length of path from the centre to the circumference and back is the diameter; and the rate of pulsation (that is, the rate of recurrence of the same phase) must for uniform motion vary inversely with the path or diameter. Accordingly if n be the number of undulations in a time-unit,

$$n = C\frac{\sqrt{\text{diameter}}}{\text{diameter}},$$

$$\text{or } n\sqrt{d} = C.$$

We should get a constant on multiplying the square root of the diameter (or radius) of a circular trough with the number of pulsations per 1′. In the following Table this is done. In column 1 are shown the values of $n\sqrt{d}$. For more palpable comparison, in column 2 are the numbers got by dividing each of column 1 by the least (which is the first).

	(1) $n\sqrt{d}$.	(2)
A . . .	2608·035	1·0000
B . . .	2613·747	1·0022
C . . .	2615·799	1·0029
E . . .	2615·397	1·0029

The close coincidence of these numbers establishes the law that the rate of wave-progression varies directly with the square root of the wave-length. The absolute velocities of progression of waves of the lengths established in the troughs are—

A . . .	63·6412	metres per 1′,
B . . .	55·8144	„
C . . .	50·0437	„
E . . .	45·3000	„

and of course these numbers, divided by the square roots of their respective wave-lengths, are constant.

According to A, a wave 1 metre long travels at the rate of

83·3060 metres in a minute.

According to B, at 82·6053 „ „
„ C, „ 82·7224 „ „
„ E, „ 83·6691 „ „

Or taking the mean, we may conclude that a wave a metre long would travel at the rate of 83·07 metres in 1′ (a little over 3 miles an hour) if it expanded circularly, and moved freely and automatically without change of wave-length.

§ 9. *Form of fundamental binodal wave in cylindrical trough.*— The beautiful smoothness and persistence of the stationary binodal waves in cylindrical vessels enables us to examine their form with some accuracy. It is at once seen that while the only part of the water free from radial motion is that in contact with the walls and the very axis of the cylinder, there is a nodal ring of constant height—which, however, is only a geometrical expression; for the water sweeps to and fro through the node radially. Putting aside for the present the internal motion of the water, let us examine the position of the nodal ring. This can be done by shifting the gauge of § 2 until there is no rise and fall on its stem, and then measuring its distance from the edge. A permanent record of the nature of the wave-system can be got by immersing a sheet of cardboard in a vertical plane passing through the axis of the cylinder and reaching to the circumference. The cardboard loses its smoothness where the water has touched it. By both of these methods it appeared that in vessel A the node was 99 millims. from the edge, in B it was 75 millims., and in C it was 64 millims. The diameters of these being respectively 595, 456, and 366 millims., the fraction of the diameters at which the nodes were formed were respectively 6·01, 6·08, and 5·72. The central amplitude was in each case 26 millims. In the vessel C this amplitude causes incipient breakers, so that the nodal point sways. It appears that in a perfect system of such waves the nodal line is $\frac{1}{6}$ of the diameter from the circumference.

§ 10. *Relative amplitude at different parts.*—When the amplitude is great, the elevation at the centre exceeds the depression at the same place. The elevation may be so great as to project water repeatedly, while the corresponding cavity is smooth and round. In such a condition the node sways. With smaller amplitudes, the alternate depression and elevation at the centre are equal to one another, and the node becomes stationary.

With great amplitudes, that at the centre is indefinitely greater than that at the edge (when the node sways). With very small amplitudes the amplitude at the edge is nearly exactly one half of that in the centre; this is the more nearly true the less the amplitude, and would, I conceive, be strictly true if the waves were conical instead of being dome-shaped.

§ 11. *Comparison with stationary waves of solids.*—A uniform elastic rod or lath, *a b c*, fig. 1, can be set in vibration in the manner shown by the dotted lines. It divides itself in such a way that $a=c=\frac{1}{2}b$ and $d=e=f$. The nodes are at $\frac{1}{4}l$ from the ends, and the amplitude at the ends is nearly the same as in the middle. A lath consisting of two isosceles triangles (fig. 2) can be thrown into similar waves whose

Fig. 1.

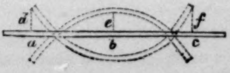
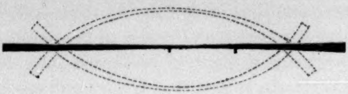


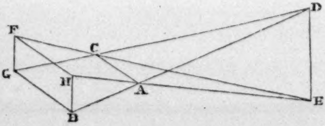
Fig. 2.



planes are at right angles to the plane of the lath (fig. 2). But the nodes are now at nearly $\frac{1}{6}l$ from the ends, and the amplitude in the middle is approximately twice that at the ends.

Imagine a thin circular board to consist entirely of such sectorial laths. They would, if strung on a circular wire passing through their centres of gravity, form a rigid system; for motion around the wire would cause gaping between the apices and crushing between the bases of the sectors. Nevertheless such motion is possible if the disk is elastic, as is shown by the formation of a nodal line around a circular plate at a distance of one sixth of the diameter from the circumference when the disk is bowed through a hole in the middle. The motion of such a disk is analogous in many respects to that of water in a circular trough. We may compare both cases with systems of isosceles triangles oscillating about lines passing through their centres of gravity. With the disk we have momentum and inertia controlled by elasticity; in the wave they are controlled by gravity. If the surfaces of the waves were conical, the following simple comparison might be drawn for an elementary displacement.

Fig. 3.



Let fig. 3, D E, be the axis of the cylindrical trough. Let D G, D B be the limiting radii of the displaced cone-sector D G B. When D sinks to E, G B will be at F H. If the sector turns on the line C A parallel to its base and one third B D from it, then D E=2F G. When D sinks to E, all the water in the tetrahedron (A C D E) leaves it, and as much water has to enter the wedge F G B A H C. These volumes must be equal. It is easy to show that they are so. This is in accordance with the general law that when two heavy planes in one plane turn together round any axis passing through their common centre of gravity, they sweep out solids of equal volume.

As the water is constrained neither to separate in the middle nor from the walls, the sectors change from conical to circular sectors and back again. And with conical waves the actual volume of water concerned in the motion is less than that traced out by the rotating sector; but this defect is the same in volume with the wedge as with the tetrahedron.

§ 12. *Level of node.*—When the wave-system is perfect, the vertical height of the node is the same as that of the surface of the liquid when at rest. The movement of particles of powder on the water shows that though the node itself is at rest, the water composing it is in violent motion. It is a stationary market in a shifting population. It shows that, as in solid and gaseous waves, a node is a region where most work is done, and accordingly where least motion ensues—a hinge.

Rectangular Troughs.

§ 13. Four troughs of zincked iron were ordered of the common depth of 1 foot, the common width of 1 foot, and the respective lengths of 1 foot, 1 foot 6 inches, 2 feet, 2 feet 6 inches. They were found to have the following dimensions in millimetres:—

	Width.	Depth.	Length.
W . . .	320	315	308
X . . .	320	315	463
Y . . .	315	315	619
Z . . .	322	315	767

About the depth, my only care was to have such a depth of water that there should be no sensible drag by friction. This appears from § 5 to be the case with about 150 millims. of water. It will be shown in § 23 that, at this width, variation in width is without sensible effect. The mean widths are given. The lengths are also the means of twelve measurements 1 inch apart. In no case was there a difference of more than 1 millim. between lengths which should have been identical.

In rectangular troughs two chief wave-systems can be established which maintain themselves automatically. They may be called binodal and mononodal respectively. The first is the counterpart of the circular waves which we have been considering. The binodal system is got by placing a wooden lath, 2 or 3 inches wide and nearly as long as the trough is wide and provided with a handle at right angles to its plane, on the surface of the water exactly halfway from either end, and depressing and raising it intact. The waves soon grow and maintain themselves for several minutes. The second or mononodal system is very easily produced by slightly tilting the trough repeatedly around one of its lower end edges. We shall consider the binodal motion first. The troughs were all filled to a depth of 260 millims. As it appears from the whole of the subjoined experiments that the number of undulations in the first minute was the same as those in the second, and so on, it followed that, as in circular troughs, the amplitude was without effect upon the frequency of pulsation.

§ 14. *Variation in length. Experiments with* W.

In 1′ there were 136 pulsations.
„ 1′ „ 135·5 „
„ 1′ „ 136 „

Mean number of pulsations in 1′ for W . . . 135·8.

In this square trough it is difficult to avoid the travelling of the nodal lines till they become diagonals.

Experiments with X.

In 1′ there were 111 pulsations.
„ 1′ „ 110 „
„ 1′ „ 110 „
„ 2′ „ 221 „

Mean number of pulsations in 1′ for X . . . 110·4.

Experiments with Y.

In 2′ there were 189 pulsations.
„ 2′ „ 189 „
„ 2′ „ 189 „

Mean number of pulsations in 1′ for Y . . . 94·5.

Experiments with Z.

In 2′ there were 169·5 pulsations.
„ 3′ „ 253·5 „
„ 4′ „ 338 „

Mean number of pulsations in 1′ for Z . . . 84·5.

Before collating these results we must examine the effect of shallowness.

§ 15. *Effect of depth on binodal undulations in rectangular troughs.*

Depth.	Number of undulations per 1′ in			
	Z.	Y.	X.	W.
millim.				
150	......	......	......	133
170	......	......	......	134
190	......	......	......	135·8
210	82	92	109	135·6
220	83	93	110	135·4
240	84	94	110·4	135·6
260	84·5	94·5	110·4	135·8
280	84·5	94·5	110·4	136
300	84·5	94·5	110·4	135·6

From this Table it appears that in rectangular troughs of the same width and different lengths, a diminution in the depth of the water takes effect in increasing the period on the longer trough before it affects the shorter one. In other words, to exhibit its normal (*i. e.* maximum) rate, a short trough need not be so deep as a longer one.

It also appears that the depth 260 millims. employed in the experiments in § 14 is sufficient.

We may therefore compare the results of § 14.

	Length.	Number of pulsations.
Z . . .	767	84·5
Y . . .	619	94·5
X . . .	463	110·4
W . . .	308	135·8

The wave-length is here, of course, the length of the trough. If the rate of progression varies directly as the square root of the wave-length, then, since the rate of repetition of phase varies inversely as the path, which is also the trough-length, we should get (as in § 8) a constant on multiplying the number of pulsations in a minute with the square root of the trough-length. This is done in the following Table, where also the quotients got by dividing all by the smallest are given:—

	$n\sqrt{l}$.	
Z . . .	2340·21	1·0000
Y . . .	2341·14	1·0004
X . . .	2375·52	1·0108
W . . .	2383·29	1·0183

These numbers are even more nearly in accord than those of circular troughs of § 8. They show that the rate of progression of a restrained wave in a trough varies also directly as the square root of its length. But the question asks itself, How is it that $n\sqrt{l}$ of straight troughs is uniformly less than $n\sqrt{d}$ of circular ones? The mean value of the constant in the circular system is 2618·652; in the rectangular it is 2360·04. These are in the ratio of 1·109 to 1. To this point I shall return in § 23.

The absolute velocities of progression of the waves are:—

Z . . .	64·8115	metres in 1′.
Y . . .	58·4955	„
X . . .	51·1152	„
W . . .	41·8264	„

If we assume the formula

$$v_1 = v_2\sqrt{\frac{l_1}{l_2}}$$

to be true for the wave-length of a metre (and we see it is approximately true for 767 millims.), it is found that a wave a metre long in a rectangular trough travels,

According to	Z,	at the rate of	74·006	metres in 1′.
„	Y,	„	74·348	„
„	X,	„	75·088	„
„	W	„	75·371	„

The mean of these is 74·7, and this is the distance which a wave a metre long would pass over in 1′ when moving in deep water between parallel walls. This is under the assumption that no velocity is lost by reflection; and the same assumption must be made in § 8 with circular troughs, where the velocity of the metre wave in circular troughs was found to be 83·1 metres in 1′.

§ 16. As the circular undulation was referred to an oscillating triangle or vibrating pair of triangular laths, so may the motion of the water above described in the rectangular trough be likened to the balancing of two rectangular laths supported at their lines of gravity, or, more closely, to the binodal vibration of a rectangular lath (see § 11). The lines of gravity are now from each end $\frac{1}{4}$ of the joint length. The nodes are at $\frac{1}{4}l$ from each end. Accordingly in rectangular troughs the nodes are at exactly $\frac{1}{4}l$ from the ends when the undulation is binodal. Also, on taking a paper section of the system (see § 9), it is found that the amplitude at the centre only exceeds that at the ends by a very small amount. As before, the node is not a line of material rest. The water sweeps through it towards the mountain which is forming at one side or the other.

§ 17. *Mononodal undulation in circular troughs.*—In circular troughs the mononodal waves can be produced by tilting or by a segmental stirrer. The former, of course, lifts the node and produces at first a longer wave than is due to the proper limits of the trough. This very fact causes the wave to accommodate itself to the trough, and to establish a nodal line which is a diameter of the cylinder at right angles to the plane of the wave. The same system is produced at once by the careful use of the segmental stirrer. Two kinds of irregularities may show themselves. The one depends upon the partial degeneration of the mononodal system into circular bi- or quadrinodal systems; the second consists of an almost unavoidable rotation of the node. Such rotation, which need not exceed a quadrant in three or four hundred waves, must of course be an experimental fault, since no reason can be assigned why it should take place in one rather than in the other direction. Neither irregularity appears to affect the rate of the mononodal system; the first gradually effaces itself probably by disaccord between the older and newer effects.

§ 18. *Effect of depth on the mononodal system in circular troughs.*

Depth.	Number of undulations per 1′ in	
	A.	B.
millims.		
145	63	76
155	......	78
165	......	79
175	65·5	80
195	......	81
225	......	82
255	71	83
280	71·5	83·5
330	71·6	83·5

§ 19. *Effect of variation in diameter on the above experiments with* A (depth = 290 millims.).

In 2′ there were 143 waves.
„ 2′ „ 143 „
„ 2′ „ 143 „
Mean rate for A per 1′ . . 71·5.

The second and third 2′ were consecutive, and accordingly the wave-rate of sequence is independent of amplitude.

Experiments with B (depth = 290 millims.).

In 2′ there were 167 waves.
„ 2′ „ 167 „
Mean rate for B per 1′ . . 83·5

Experiments with C (depth = 185 millims.).

In 2′ there were 183 waves.
„ 2′ „ 183 „
Mean rate for C per 1′ . . 91·5.

Experiments with E (depth = 870 millims.).

In 1′ there were 103·5 waves.
„ 1′ „ 103·5 „
„ 1′ „ 103·5 „
Mean rate for E per 1′ . . 103·5.

Putting the results together—

	Diameter.	Mean n.	$n\sqrt{d}$.	Depth. millim.
A	595	71·5	1739·09	290
A′	595	71·6	1741·53	330
B	456	83·5	1782·06	290
C	366	91·5	1760·49	185
E	300	103·5	1789·62	870

It appears both from § 18 and from the above, that A or even A′ was scarcely deep enough to give the normal rate for their great wave-lengths. It also appears that for the shorter system of C the depth 185 is insufficient. From § 18 it seems that the maximum or normal rate is reached. We may take the nearly identical numbers for B and E for the valuation of the mean constant, which is 1785·84 (in metre-minute measurement).

§ 20. *Mononodal undulations in rectangular troughs.*—In rectangular troughs the mononodal wave-system can be established by tilting or by the use of a wide stirrer. The wave-length is twice that of the trough. As the shallowness influences longer waves more than shorter ones, we must look for greater discrepancies here than in §§ 15, 16, 17. Taking the same troughs, namely Z, Y, X, W, the first experiments were to determine the influence of depth.

Influence of Depth on Mononodal Waves in Rectangular Troughs.

Depth.	Z.	Y.	X.	W.
millims.				
120	......	......	......	92
140	......	......	......	93
160	......	......	......	94
180	......	......	......	94
200	49	......	......	94
220	51·5	......	......	94
240	53	61	74·6	94
260	54	62·2	75·2	94
280	55	63	76	94
300	55	63	76	94

Experiments with Z (depth =290 millims.).

In 2′ there were 110 waves.
„ 3 „ 165 „
Mean rate for Z per 1′ . . . 55

Experiments with Y.

In 3′ there were 189 waves.
„ 2′ „ 126 „
Mean rate for Y per 1′ . . . 63

Experiments with X.

In 3′ there were 228 waves.
„ 2′ „ 152 „
Mean rate for X per 1′ . . . 76

Experiments with W.

In 1′ there were 94 waves.
„ 1′ „ 94 „
„ 1′ „ 94 „
Mean rate for W per 1′ . . . 94

Collecting and multiplying each number of waves by the square root of its trough-length (trough-length = ½ wave-length, the ratio is the same), and dividing all such products by the least of them, we get the following numbers :—

	Length of trough. l.	Mean number of waves in 1′=n.	$n\sqrt{l}$.	
Z . . .	767	55	1523·21	1·00
Y . . .	619	63	1568·42	1·03
X . .	463	76	1635·32	1·07
W . .	308	94	1649·70	1·08

In spite, therefore, of the apparent attainment of maximum rate at 280 millims., we must reject the two longest waves in adopting a constant, which, taking the mean of the two shorter systems, is 1642·5.

In all the four cases, namely mono- and binodal waves in circular and rectangular troughs, the value of $n\sqrt{l}$ or $n\sqrt{d}$ increases as the length of l or d decreases. The increase of rate of wave progress due to increased wave-length is somewhat more than counterbalanced by the increased path, so that the "constant" is larger with smaller troughs.

§ 21. *Comparison between mononodal and binodal waves in rectangular troughs.*—In the same trough the wave-length of the mononodal system is always that of the binodal system, the former being $2l$ ($l=$ trough-length). If the ratio of progression be as the square root of the wave-length, then the number n of waves in a given time, varying directly as the rate of progression and inversely as the distance, in the same trough

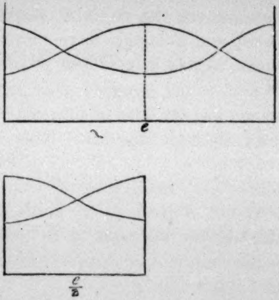
$$\frac{\text{number of binodal}}{\text{number of mononodal}}=\sqrt{2}.$$

How far this is borne out in the experiments is seen in the following comparison of the shorter troughs, where the mononodal system is normal.

	Length of trough.	$\sqrt{2}\times$mononodal.	No. of binodal.
X . . .	463	107·47	110·4
W . .	308	132·94	135·8

§ 22. If we take a binodal system and insert a rigid plane diaphragm vertically down the middle and at right angles to the plane of the wave, we divide it under little disturbance obviously into two mononodal systems of half the trough-length, but of the same wave-length as before. These should oscillate (inde-

Fig. 4.



pendently of one another) at the same rate and at the rate of the original system. The experiment which illustrates this relation most directly is Y(l=619 millims.) of the binodal, where n=94·5, and W (l=308) of the mononodal, where n=94·0. Such a material diaphragm cannot satisfy the ideal conditions; for if at rest, additional friction is introduced by the motion along its surfaces of pairs of particles at each side exerting equal and opposite pressures. These, in the absence of the diaphragm, exert no friction on one another, since they move at the same rate.

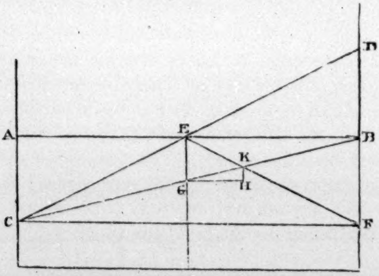
§ 23. *General comparison with Pendula.*—As the amplitude of the wave produces no sensible effect on its rate of progression, and the latter varies for small wave-lengths directly as the square root of the wave-length both in circular and rectangular troughs, the oscillation of water may be closely compared with that of a pendulum, which for small circular arcs is isochronous, whatever be the amplitude of displacement; height of wave : height of pendulum-bob=$v(1-\cos\frac{1}{2}\theta)$. The mean velocity of the pendulum-bob varies as the arc for the same radius; for the same small angular displacement inversely as the square root of the radius. We have then the analogous cases:—

$$\begin{array}{ll}
\text{Pendulum.} & \text{Water-wave.} \\
n\sim\dfrac{\text{velocity}}{\text{path}}, & n\sim\dfrac{\text{velocity}}{\text{path}}, \\
\quad\sim\dfrac{\sqrt{r}}{r}, & \quad\sim\dfrac{\sqrt{l}}{l}, \\
\quad\sim\dfrac{1}{\sqrt{r}}, & \quad\sim\dfrac{1}{\sqrt{l}}.
\end{array}$$

The moving force in a simple pendulum is always equal to its weight, though this force is always applied partly to the point of suspension. In the moving water the to-and-fro motion is due to the action of a variable difference of pressures (heights). The law of oscillation, however, is preserved, although the undulations are the result of particle-motion in the mass of the liquid, and although these particles in the case of stationary waves, as was pointed out so many years ago by the brothers Weber, do not complete their orbital motion, but themselves swing, like the bobs of elementary pendula, to and fro in short-arc'd rhythmic sequence. Whenever the momentum and inertia vary together (as in all cases of solid magnification—*i. e.* alteration of size, conservation of shape, and either conservation or alteration of density), the pendular law must be preserved, and this whether the mass to be moved is wholly active, as in the pendulum itself, or more or less passive, as the logan oscillation of a common balance.

In rectangular troughs it is easily shown that, if the surface is plane, the pendulum (elasticity) or torsion law must hold good, and that the force of restoration is proportional to the linear measure of disturbance. If A B (fig. 5) be the level of the water in a rectangular trough and it is set in mononodal undulation, so that in one phase the surface is at C E D, the line B D, B F, or A C is half the amplitude. Consider all the water below C F to be without influence. The rectangle of water A C B F has become the triangle C F D. Join C B and E F; their intersection K is the centre of gravity of C D F, and is $\frac{2}{3}$ of C B from C, &c. Draw G H horizontal and K H vertical till they intersect. The centre of gravity of the rectangle A C B F has been raised by the change of form from G to K—

Fig. 5.



that is, from H to K. The work in the displaced water is $DCF \times KH$.

$$KH = KG\frac{BF}{BC},$$

and

$$KG = (\tfrac{2}{3} - \tfrac{1}{2})BC = \tfrac{1}{6}BC,$$

or

$$KH = \tfrac{1}{12}DF.$$

The work done and potential at maximum excursion vary directly with displacement. What is true for a plane is of course true for a rectangular trough. It can also be shown to be true for disturbances in circular troughs from fig. 3, § 11, and follows indeed without further proof from the equal tetrahedron and wedge having been generated by the rotation of a triangle balanced about one of its lines of gravity. The fact that the surface of both wave-systems is curved, and that this curve involves the cohesion and viscosity of the liquid, puts the exact solution of the question beyond the present power of analysis.

§ 24. *Absolute comparison of circular waves with pendulum.*—It appears that both the binodal waves in circular troughs and the binodal waves in rectangular troughs follow the pendular law, and that the mononodal waves in rectangular and circular troughs do so nearly. It follows from § 23, where it is shown that no dead weight has to be stirred, that, if the liquid has no viscosity, not only should the pendular or torsion law hold good, but the actual rate of undulation in the binodal system in the unhampered or circular wave should be the same as that of a pendulum of radius equal to half the wave-length—that is, to the diameter of the cylinder. If a pendulum of length l be suspended over a circular trough of radius l, it keeps time very nearly with the fundamental undulations in the trough. This can be beautifully shown in an experiment where the pendulum-length can be altered till it is isochronous with the pulsations. But it appears best from the data already given. Taking the length of the seconds-pendulum at London at 994 millims., the number of oscillations which pendula of the radii of the circular troughs give is shown in the following Table,

$$n' = 60\sqrt{\frac{994}{r}}.$$

	(1) Radius of trough, r.	Number of liquid undulations.	(n'). Number of oscillations of pendulum of length r.
A …………	297·5	106·9	108·6
B …………	228	122·4	125·3
C …………	183	136·7	139·4
D …………	157	149·0	150·9
E …………	150	151	154·4

The pendulum-oscillations exceed the liquid ones, on an average, 2·8; on the corrected average, under the omission of E, 2·05. The mean per-cent. deficit is about 1·5. Considering that the liquid undulations do not, even in the largest troughs, sustain themselves for more than 20′, while a pendulum in air will oscillate for hours, and remembering that rate of decadence is nearly proportional to individual delay, I do not think that the above numbers are at all in disaccord with the assertion that the *fundamental or binodal circular undulations in an infinitely deep circular vessel are isochronous with those of a pendulum whose length is equal to the radius of the vessel.*

In hemispherical and cycloidal vessels I anticipate that a still nearer identity would be obtained. The exact form of the cup of revolution which would give the nearest approach to the pendulum-rate depends of course upon the shape of the wave; and here we are again referred to the complex influence of viscosity and cohesion.

§ 25. *Comparison of rectangular troughs with pendulum.*—Making a similar comparison to the above with rectangular troughs having binodal undulations, let us find what are the lengths of pendula isochronous with the water-waves in the troughs Z, Y, X, W respectively. The number of liquid undulations being n, the formula

$$l=\left(\frac{60}{n}\right)^2 994$$

gives us the required lengths of the pendula in millims.

	Trough- or wave-length.	Number of liquid undulations.	Length of isochronous pendulum.	$\frac{2}{\pi}\times$ trough-length.
Z ………	767	84·5	501·194	488·4
Y ………	619	94·5	490·482	394·1
X ………	463	110·4	293·627	294·8
W………	308	135·8	194·028	196·1

It appears, therefore, that *the binodal undulations in a rectangular trough are isochronous with the oscillations of a pendulum whose length is* $\frac{2}{\pi}$ *of the trough or wave-length.*

§ 26. *Comparison between circular and rectangular troughs in binodal vibration.*—Briefly, it has appeared that the differences between the wave-systems in circular and rectangular troughs, both being binodal, are as follows:—In rectangular troughs the nodes are one fourth of the trough-length from the ends. In circular troughs they are one sixth of the diameter. In rectangular troughs the amplitude is a very little more at the centre than at the edges. In circular troughs it is very little more than double that at the edges. The rate of progression in both varies as the square root of the wave-length; and the rate of sequence, or n, varies inversely as the square root of the wave-length. The waves of the circular system are isochronous with a pendulum of half the wave-length; in the rectangular trough this pendulum must be $\frac{2}{\pi} \times$ wave-length. If reflection does not alter velocity, the circular wave 1 metre long will travel 83·1 metres in one minute if it preserves its wave-length; while a wave 1 metre long between walls will only travel 74·7 metres in one minute. In answering the question as to whence this latter difference arises, a few preliminary experiments are useful. That waves in circular troughs endure longer than waves of the same amplitude in rectangular ones may be attributed in great measure to friction. That element of friction which is due to adhesion and cohesion can be directly compared as follows. In a square trough in one complete undulation the surface rubbed by the two end lines of particles is $4al$ if a is the amplitude and l the trough or wave-length. The surface rubbed by the two side-lines of particles is $2al$. The whole surface rubbed by the four edges of the top layer of particles is $6al$. In the circular trough whose diameter is equal to l, the amplitude at the centre being a, the surface rubbed by the edge ring of particles in a complete undulation is $2a\pi\frac{1}{2}l$ or $a\pi l$; or the cohesive friction on the square trough is for the upper layer 1·9 times that of the circular trough for the same wave-length and edge-wave amplitude. If such friction were the cause of the slower rate of wave-progress in rectangular troughs, we should expect the effect to be less in wider troughs than in narrower ones, because, while the surface scraped would be the same, the mass of the water would be greater.

Accordingly the rectangular troughs W, X, Z, which have the dimensions

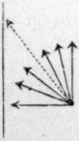
	Width.	Length.
W . . .	320	308
X . . .	320	463
Z . . .	322	767,

were filled to a depth of 280 millims., and by means of long laths set in binodal undulation in the direction of their widths. The wave-lengths of W and X were accordingly identical; and the wave-length of Z was only 4 millims. longer. The mean of four concordant determinations in each case showed in 1′,

for W . . .	134
„ X . . .	134
„ Z . . .	133·5

In brief, the length of the trough *across* which the system was established produced no sensible variation in rate of phase-recurrence, and therefore presumably none in wave-progression.

Fig. 6.



But a wave in a rectangular trough undergoes a constraint quite other than that of friction. Its motion is repressed by the sides. It presses equally and radially in all directions. The wall offers resistance perpendicular to itself. The resultant of the quadrant of the forces in fig. 6 makes an angle of 45° with the wall. The lateral pressure on the wall is $\frac{1}{\sqrt{2}}$ of the total radial pressure; the onward pressure is also equal to $\frac{1}{\sqrt{2}}$, the total pressure. The constant obstruction thus offered by the walls is the cause of the preservation of amplitude. The constant obstruction thus offered by the rigid parallel walls, while preserving amplitude, is, I imagine, the main cause of the lesser rate of wave-progress in rectangular troughs; and could exact measurement be effected, I should expect that the ratio between the times of subsidence from one amplitude to a lower one in a circular and rectangular trough would be found to be nearly the ratio of 83·1 to 74·7. The measurements of rate of subsidence cannot be made of sufficient exactness to test this experimentally; but it is clear enough that binodal waves in circular troughs are far more enduring than those of rectangular ones.

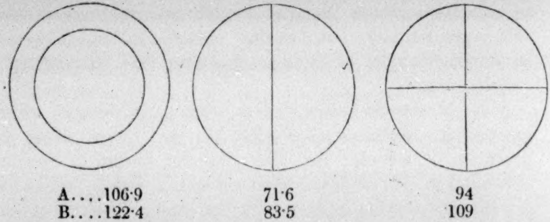
§ 27. It may be shown that in rectangular troughs, when mononodal undulation takes place, the path of the centre of gravity is a parabolic arc, supposing the surface to remain flat. But as this condition never obtains in reality, and as no one particle remains in its motion the centre of gravity of the moving mass, this parabolic motion of the centre of gravity is of no direct account.

§ 28. *Cross binodal undulations in circular troughs.*—On pressing simultaneously two opposite points of the rims of A and B, they yield sufficiently to start undulations which are analogous to the ring of a bell or the fundamental quadrant vibration of a Chladni plate. By repeating the push at proper intervals, a well-shaped system of undulation is established, consisting of quadrant waves separated by diametral nodes at right angles to one another. The rate of recurrence of these in troughs A and B was examined. The troughs were filled with 300 millims. of water.

	Number of recurrences of same phase in 1′.
A.	94 94 94
B.	109 109 109

Taking the troughs A and B, we have the following comparison :—

Fig. 7.



§ 29. *Compound motions.*—The motion given to a glass of water when the axis is made to describe a narrow cylinder and then brought to rest, whereby a wave careers round the vessel, is a modification of the mononodal. If the node of such a system performs one complete revolution during the phase-life of the mononodal wave, the crest reappears in the same place. In the trough B, which held 300 millims. of water, in each of three experiments the crest reappeared in the same place eighty-four times in one minute. The travelling of the node, therefore, makes no difference in the wave-rate. The motion of the surface resembles that of a disk which is at right angles to an axis when the axis describes a circular cone.

If the water revolves on its axis in the trough, mononodal undulations are carried round in a similar manner.

The revolution of the water does not interfere with the period of the binodal circular waves. Thus the water in A, being 300 millims. deep, was stirred round; and while in rapid rotation, it was impressed with the circular binodal wave-system. In each of two minutes 107 waves were counted; and the last minute was immediately following the first. At the end of the experiment the water was still circulating at the rate of 12° per 1″.

§ 30. *Oil on water.*—The restraining effect of a layer of oil on the water, inasmuch as it causes the more rapid loss of amplitude, might be supposed to effect a retardation in the wave's progress. Such is not the case, apparently; for a layer of olive-oil 5 millims. in thickness undulated with the water beneath it in the trough B at such a rate as to give in three experiments the numbers 123, 122·5, 122·5.

§ 31. *Viscosity.*—Glue added in varying quantity up to 1 lb. of the solid in the trough B did not change the rate from 123.

§ 32. *Images on wall.*—It can readily be imagined that the sun shining on water or, better, water covered with oil in one of the states of motion above described, gives most elegant figures on the wall. The circular binodal, for instance, shows lateral and vertical loop-exchange; the cross binodal circular shows a collapsing and expanding parallelogram; the binodal rectangular exhibits bars of light which approach and recede with a pause and slight regression. From the distribution of light in the images, the kind and, to some extent, the degree of curvature can be ascertained. Thus, when the circular binodal has nearly subsided, the only alteration visible is the change of luminosity in the centre, due to the alternating convexity and concavity.

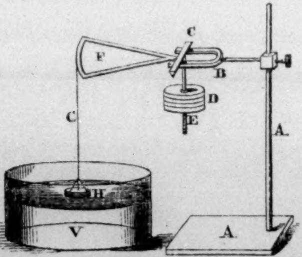
§ 33. *Recent contributions to the subject.*—On the occasion of the communication of my experiments to the Physical Society, I invited the Members to offer suggestions regarding the *rationale* of some of the stationary waves which were there exhibited.

Mr. O. Lodge has been good enough to point out how completely the phenomenon depends upon the relationship between wave-length and rate of wave-progress. I regard the results as being of value, in a great measure because they show, I believe, more conclusively than can be shown by the wave-progress on open sheets of water, that this law, that the velocity of progress is directly proportional to the square root of the wave-length, is a true one. This they do by reason of the conservation of wave-length.

In 'Nature' of July 28th Lord Rayleigh discusses the theoretical aspect of the question, and shows that the wave-rate of sequence which I had given for circular binodal waves corresponds closely with the theoretical number deduced from the hypothesis that in troughs of infinite depths the variation of the diameter is the only thing which affects the wave-rate or period. That the pendulum-, or, as I should prefer to call it, the Logan-law, or law of magnification, should be so closely preserved through the complex molecular movement which actually takes place is of the greatest interest—the more so as the paths of the particles are not increased by an increase of the trough, but there is only a diminution of the eccentricity of their fragmentary orbits. This has, of course, its counterpart in vibrating elastic rods. The mathematical interpretation of the experimental fact is very valuable. I do not suppose that such interpretation can be complete until account is taken of the cohesion and viscosity of liquids, to which, on the other hand, such experiments may supply a new weapon of attack. The other kinds of wave-systems, the rate of which Lord Rayleigh predicts mathematically, and which in rectangular troughs were described, but not investigated with regard to their period, in the elaborate work of the brothers Weber, I had already examined in regard to their period both in circular and rectangular troughs. In this respect it is safe to predict that every system of vibration of a square or round Chladni plate may be reproduced in a square or round vessel of water, and that the time-ratio will be preserved. Further, in water-wave systems we can have mononodal vibration, which for automatic motion is inadmissible in vibrating plates. With the exception of those above studied, the more complex systems appear short-lived.

34. That the wave-motion of the binodal circular system is not only isochronous with the radius pendulum, but also coincides with pendular motion throughout the whole course of both, appears from the fact that the two sustain one another when in direct integral mechanical connexion. Fig. 8 shows such an arrangement. A tuning-fork, B, having the plane of its prongs horizontal, is fastened to a heavy stand, A. Resting

Fig. 8.



on the fork is a knife-edge, C. To C is attached a vertical screw-rod, E, carrying a nut (invisible in the figure) which supports the heavy weight D, consisting of a series of leaden disks. Also fastened to C is a cardboard sector, F. The circular edge of F is split, and into the crack is pasted a paper gutte. A silk thread, G, is fastened to F, and carries a little paraffin disk, H. The pendulum-length is altered by the nut E until the disk H is isochronous in its motion with the water in V. The vessel V is shown out of proportion small. When the right length of E is obtained, the disk H moves up and down with the water without a ripple for twenty minutes or more, showing how truly pendular is the wave-motion. It follows that, for small disturbances, the rate of vertical motion at the normal level must be closely proportional to the altitude of the wave or amplitude of undulation.

XXIII. *On the Flow of Electricity in a uniform plane conducting Surface.*—Part II. *By* Professor G. Carey Foster, *F.R.S., and* Oliver J. Lodge, *B.Sc.*[*]

43. In an unlimited uniform conducting surface, at given points of which there are sources and sinks of electricity, the electrical condition of each point within a limited area must be determined by the electrical condition of its boundaries. Moreover, if an equal similar area be taken in a *limited* conducting sheet, whose conductivity and thickness outside this area vary in any manner, the electrical state within the area will still be the same as in the previous case, if only the conditions at the boundaries remain unchanged; for evidently the electrical flow within the area will depend on the actual electrical state of the boundaries, and not upon the way in which this state has been produced. Consequently the truth or falsehood of conclusions arrived at, like those given in Part I. of this communication, on the assumption of an unlimited uniform conducting surface, may be tested by experiments on a conducting sheet of finite size, if we can ensure that, at the edges, the electrical conditions are the same as they would be at the boundaries of a portion of the same

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[*] The substance of this paper was read to the Physical Society on the 27th of February, 1875; but it contains also experimental results obtained since that date.

size and shape taken in an infinite sheet in which there is a known distribution of sources and sinks.

44. In order to carry out such experiments, it is needful that the boundaries of the part of the conducting sheet examined should be formed either wholly by equipotential lines, or partly by equipotential lines and partly by lines of flow. It is evident that an area within which there is a flow of electricity (unless the circuit is complete within this area, as in a conducting plate moving in a magnetic field) cannot be bounded entirely by lines of flow; for in that case no electricity could enter or leave the area.

Any line in a conducting sheet can be practically made to be an equipotential line by laying along it, and in close contact with the sheet, a conductor whose resistance in directions parallel to the sheet is insensible—for instance, a thick band of copper. And any line along which the sheet is bounded by non-conductors (*e. g.* the free edges of the sheet) is necessarily a line of flow, since no electricity can pass across such a boundary in either direction. The experimental conditions can thus be chosen so that a conducting area may be bounded either by equipotential lines wholly, or partly by equipotential lines and partly by lines of flow, placed so as to correspond to any given system of poles.

45. Methods of testing experimentally the conclusions deduced from theory, founded on the general principles above indicated, have been employed by various investigators. Thus Kirchhoff, in his paper of 1845 (Pogg. *Ann.* vol. lxiv.), traced the form and distribution of the equipotential lines on thin sheets of metal bounded by four circular arcs, two of which, being free edges, were lines of flow and belonged to the same circle; while the others, of much smaller extent than the first two, were equipotential lines determined by the contact of small cylindrical electrodes (conducting wires) with the margin of the sheet. He traced the course of the equipotential lines by fixing a wire connected with one terminal of a galvanometer in succession at several points arbitrarily chosen, and in each case finding, by trial with a wire from the other terminal, a sufficient number of points having the same potential as the selected point. To ascertain the distribution required to make the difference of potential between consecutive lines constant, he determined a number of points along the straight line joining the electrodes, such that the difference of potential between each one and the next was just

sufficient to compensate the constant electromotive force of a thermoelectric element. The form of the equipotential lines in some rather more complicated cases was determined in the same manner by Quincke[*], and in other cases again by students in the Edinburgh University Laboratory[†]. Schwedoff[‡] also has investigated the direction of the flow at various points in a rectangular sheet of tinfoil.

Recently an elaborate series of experiments, by a method similar to Kirchhoff's, has been carried out by Professor W. G. Adams[§], who has also extended the same method of inquiry to conductors of three dimensions by the employment of conducting liquids (mercury, sulphate of copper, sulphate of zinc).

Verifications by Kirchhoff and by Mach of the theoretical expressions for the strength of the current at different parts of a conducting sheet have already been referred to (see Part I. § 15, footnote.

Experimental measurements of resistance, in addition to the attempts of Kirchhoff already mentioned (Part I. § 1), have been made by Gaugain[||], who measured the resistance of a solution of sulphate of copper contained between two circular copper cylinders placed one inside the other with their axes parallel, and thus verified Blavier's formula (see Part I. § 24, footnote)—by v. Obermayr[¶], who verified Kirchhoff's formula for circular disks, and also a formula deduced by Stefan for the resistance of an oblong strip with circular electrodes placed on the central line, by experiments with disks and strips of platinum-foil—and by Domalip[**], who measured the resistance of thin circular layers of a solution of sulphate of zinc contained between parallel glass disks, the electrodes being zinc rods inserted through holes in the upper piece of glass.

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[*] Pogg. *Ann.* (1856) vol. xcvii. p. 382.

[†] W. Robertson Smith, Proc. Edinb. Roy. Soc. 1869–70, p. 96.

[‡] Schwedoff (Pogg. *Ann.* 1873, Ergänzungsb. vi. p. 85) inferred the direction of the resultant flow at any point of a conducting sheet in which there are two equal opposite poles, from simple geometrical considerations, to a great extent similar to those employed by Professor W. R. Smith and by ourselves (Phil. Mag. vol. xlix. p. 394). We should have referred to his paper before, if we had known of its existence.

[§] Proc. Roy. Soc. (1875) vol. xxiii. p. 280.

[||] *Ann. de Chim. et de Phys.* [3] vol. lxiv. p. 200 (1862).

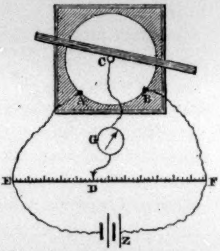
[¶] *Wien. Akad. Ber.* (1869) vol. lx. pt. 2. p. 245.

[**] Carl's *Repertorium für Experimental-Physik*, 1874, vol. x. p. 23.

46. The results of all these investigations are in such close agreement with calculation that it cannot be considered that any further confirmation of the general theory is needed. It nevertheless seems worth while to point out, and to illustrate by specimens of the results obtained by it, a simple method which serves both for determining the form and distribution of the equipotential lines on a conductor of two dimensions, and for measuring the resistance of the part of it contained between any two given lines. In all except the very simplest cases, it would be much easier to determine the equipotential lines and resistance experimentally in this way than by the corresponding calculation.

47. The method referred to is an application of the principle of Wheatstone's bridge, and will be intelligible from fig. 1. The poles of a battery (Z) are connected with the two ends of a graduated German-silver wire, E F, and also with two fixed points, A and B, of the conducting disk on which the equipotential lines are to be traced; a galvanometer, G, is connected to a sliding contact block D on the wire E F, and to a contact-pin C, with the point of which the sheet can be explored. Under these circumstances the points C and D will be at the same potential whenever the galvanometer shows no deflection; hence if the pin C be moved about on the disk, and a mark made at every position for which the galvanometer is not affected, all these marks will lie on one equipotential line. By repeating this operation, after shifting the slider D along the graduated wire, another equipotential line can be marked out on the sheet; and any number of lines can be traced, so as to correspond to equal differences of potential, by shifting the slider through successive equal distances.

Fig. 1.

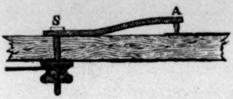


48. In the actual experiments the connexions were not made exactly as shown in fig. 1, but the positions of battery and galvanometer were interchanged. This arrangement made it unnecessary to use a separate key for closing the battery-circuit, as the circuit was completed by the contact of the pin C with the

sheet; and since these contacts were only momentary, no sensible disturbance can have arisen from unequal heating. The battery used was a single cell of Smee; the galvanometer was a reflecting one of low resistance. The disk was a circular piece of tinfoil, of 27·8 centims. diameter, pasted on the bottom of a tray or shallow box of hard wood, on the edges of which rested a wooden bar supporting the contact-pin C. This last slid in a vertical tube fixed to the wooden bar; a spiral spring inside the tube kept the pin out of contact with the sheet except when it was pressed down. When the right position on the sheet had been found by means of the blunt point of the pin, this position was marked by pressing the pin down harder, so as to make a slight indentation in the tinfoil.

The mode of making contact at A and B which we found to answer best is shown in fig. 2. A S is a brass spring with a projecting pin A, and a screw S with nut and washer. The screw passes through the wood of the tray and is fixed by screwing up the nut, the conducting wire being clamped between the nut and washer. The rounded end of the pin A rests on the tinfoil surface, pressing it firmly and making a good circular contact. It was found necessary to lubricate the bar carrying C with blacklead to prevent its causing vibrations when sliding on the edges of the tray, and so deranging the contact of the springs A and B.

Fig. 2.



49. After a series of points have been marked on the tinfoil in the manner described, a line may be drawn through each set by hand. A set of lines obtained in this way for the case of two equal opposite poles on the edge of a circular disk correspond to a portion of the diagram in Plate IV. bounded by a complete line of flow. P being any one of the dots marking such a line, if they are circles with the poles as inverse points, the ratio $\frac{AP}{BP}$ will be constant for any one line (Part I. § 16); and if they are at equal differences of potential, the values of this ratio will vary from one line to the next by a constant factor μ (Part I. § 18). The numbers given in the following Table show to what extent these requirements are fulfilled by the ex-

perimental lines. Nine or ten points on an average were determined for each line; but as a specimen of the results it seems sufficient to give the ratios found for four points only, viz. (in the case of each line) for a point near either end, a point on or near the straight line joining the poles, and a point about halfway between this and the more distant end of the equipotential line.

Values of $\frac{\mathrm{AP}}{\mathrm{BP}}\ (=\mu^n)$ *for four different points on each of eight different equipotential lines.*

	μ^{-3}.	μ^{-2}.	μ^{-1}.	μ^0.	μ.	μ^2.	μ^3.	μ^4.
	·200	·335	·568	·996	1·71	2·94	5·05	8·75
	·200	·327	·568	1·00	1·70	2 95	5·13	9·01
	·190	·323	·573	1·00	1·71	2·98	5·18	8·97
	·196	·333	·573	1·02	1·73	2·98	5·53	8·91
Means.	·1965 $=(1{\cdot}72)^{-3}$	·3295 $=(1{\cdot}74)^{-2}$	·5705 $=(1{\cdot}76)^{-1}$	1·005	1·71 $=(1{\cdot}71)$	2·96 $=(1{\cdot}72)^2$	5·22 $=(1{\cdot}73)^3$	8·66. =(1· 7)

In the experiments by which the above values were obtained the distance between the poles was 19·45 centims. Taking this as the value of $2a$, and 1·724 for μ, a system of coaxal circles was drawn by the method of § 21, Part I. A tracing of the experimental curves was then superposed on these theoretical circles; and the two sets were found to coincide almost exactly.

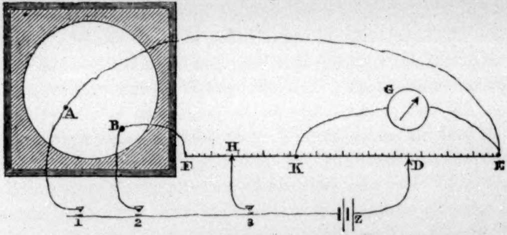
Another case tried was that of a circular disk with four equal poles on its edge, two positive and two negative; the equipotential lines obtained experimentally agreed in all particulars with those drawn by the superposition method of § 35.

50. The arrangement of fig. 1 serves not only to lay out the equipotential lines on a disk, but also to determine its resistance; for if the pin C be kept in contact with any point of the disk (with the electrode A for instance), while D is adjusted so that the galvanometer stands at zero, and if C is then moved to another point (say the electrode B) and D again adjusted, the resistance of the part of the disk between the equipotential lines which respectively pass through the two positions of C is the same fraction of the resistance of the branch E A B F, as the resistance of the part of the wire between the two positions of D is of the resistance of the whole wire E F. Accordingly the resistance of the disk is given, in terms of that of unit length of the wire E F,

as soon as the ratio of the resistances E A B F and E D F is known. In order to determine this ratio, the connexions shown in fig. 1 were somewhat modified, the arrangement finally adopted being that shown in fig. 3.

One pole of the battery is connected by means of a slider D with some point of the wire E F, while the other can be connected at pleasure, on the one hand, with either of the electrodes A and B upon the disk by pressing one of the contact-keys 1 and 2, or, on the other hand, with another slider H on the wire E F by pressing the key 3. The galvanometer is permanently connected with one end E of the wire, and with a fixed point K between the sliders D and H. The part of the graduated wire between the galvanometer-terminals E and K thus takes the place of the whole length E F in the arrangement shown in fig. 1. To measure the resistance between A and B in terms of that of the graduated wire, it is then only necessary to

Fig. 3.



press successively the keys 1 and 2, and each time to adjust the slider D so as to give no deflection on the galvanometer, and then to connect the battery with H, by the key 3, and to find the positions of D required to keep the galvanometer at zero for two positions of H separated by the known amount h. If we denote by s the distance through which D has to be shifted on the wire E K when the battery-contact on the disk is transferred from A to B, and by d the shift of D corresponding to the shift of H over the distance h, we have

$$\frac{\text{resistance between A and B}}{\text{resistance of } s} = \frac{\text{resistance of } h}{\text{resistance of } d};$$

and since s, h, and d are all of them lengths measured on the same wire, their resistances are simply as their lengths. Hence, taking as unit of resistance the resistance of unit length of the graduated wire, we have

$$\text{resistance A to B} = s\frac{h}{d}.$$

It will be observed that the ratio $\frac{h}{d}$, which we will hereafter denote by $\frac{1}{\epsilon}$, is the same as that of the resistances E A B K and E D K.

51. In all measurements of resistance it is necessary, not only that the contact of the electrodes A and B with the disk should be constant and easily reproduced, but also that the area of contact should be measurable; the difficulty of realizing these conditions with accuracy is by far the most important source of error to which the determinations of resistance are liable. In order to obtain a measurable and constant surface of contact, the spring contact-pieces employed in the experiments for plotting the equipotential lines were replaced by the following arrangement. A thin cylindrical rod of brass, to which two wires could be attached, slid vertically in a V-groove down the side of a rectangular wooden block. The lower end of the rod was filed accurately flat and perpendicular to its length, so as to touch the tinfoil with its entire surface. Constant pressure against the tinfoil was maintained by placing a weight on a cap fixed to its upper end. Two such rods constituted the electrodes A and B; and tolerably constant contacts could be made with them after a little practice. Nevertheless individual readings are apt to vary a good deal, and it is necessary to take the mean of several distinct experiments in order to get a single trustworthy result.

The numbers given in the annexed Table will serve to illustrate the variations occurring among single readings. They refer to a set of experiments in which the electrodes were placed approximately at the ends of a diameter of a circular disk.

	$2a$.	s.
	26·30	69·4
	26·22	69·7
	26·30	68·3
	26·30	68·3
	26·00	67·5
	26·31	68·7
	25·95	69·0
	26·25	68·6
	26·30	69·8
	26·00	69·4
	26·25	67·9
	26·31	68·7
	26·05	69·2
Means	......26·20	68·80

52. The principal results of the measurements of resistance made by the method explained above are given in the Tables which follow, in such a way as to show how far they agree with the values indicated by theory. Every entry in the Tables is a mean value deduced from the number of experiments stated in the first column; the last column in each Table contains a quotient which theoretically should be constant. Common logarithms are employed, except when it is otherwise stated. The symbols used in the headings of the columns have the following significations:—

R, resistance in terms of an arbitrary unit (=·00155 ohm; being the resistance of one centimetre of the wire E F).

x, resistance of the contacts between the electrodes and the sheet (found by special experiments).

ϵ, ratio of the resistances EDF and EABF (figs. 1 or 3), or $\frac{d}{h}$ (§ 50).

s, shift of the slider D.

$2a$, or AB, distance between the poles.

ρ, radius of the circle of contact of the electrodes, or half the diameter of the brass rods.

r, radius of the tinfoil disk.

δ, thickness of the disk.

κ, specific conductivity of the material of disk.

L, the expression $\log_{10}\frac{AG \, . \, BF \, . \, A'G \, . \, B'F}{\rho^2 \, . \, A'F \, . \, B'G}$ (see equation (20a), § 39, Part I.).

C, the constant $\frac{\log_e 10}{2\pi\kappa\delta}$.

The following relations hold good among the above quantities

$$\mathrm{R}+x=\frac{s}{\epsilon}\ (\S\ 50)$$

and

$$\mathrm{R}=\mathrm{CL}\ (\S\S\ 38 \text{ and } 39, \text{ Part I.}),$$

whence

$$s-\epsilon x=\epsilon\mathrm{CL}.$$

First set of Resistance experiments, April 1874.

ϵ was practically constant and $=5{\cdot}147$, $\rho={\cdot}163$, $\epsilon x=2{\cdot}42$.

Case 1. Two equal electrodes on the edge of a circular disk (Plate IV.).

Number of experiments.	r.	$2a$.	s.	$\dfrac{s-\epsilon x}{\log\frac{2a}{\rho}}$.
13	13·16	26·20	68·80	30·1
13		18·83	65·42	30·5
9		15·465	62·08	30·0
12		13·466	59·47	29·8
16		10·185	57·05	30·6
17	7·62	15·224	61·66	30·1
6		11·675	58·02	30·0
10		10·35	57·20	30·4
5		8·78	56·60	31·3
		Weighted mean......		...30·29

In this case $\mathrm{L}=4\log\dfrac{2a}{\rho}$ by formula (13), §§ 25 & 39, Part I.,

and therefore $\dfrac{s-\epsilon x}{\log\frac{2a}{\rho}}=4\mathrm{C}\epsilon=\text{const.}$: the mean value 30·29 gives

$2\mathrm{C}=2{\cdot}943$.

Case 2. Two equal electrodes, one at the centre and one on the edge of a circular disk.

Number of experiments.	r.	s.	$\dfrac{s-\epsilon x}{\log\frac{r}{\rho}}$
8	13·16	46·47	23·10
7	7·62	41·81	23·60
		Mean	......23·33

Here $L=3\log\frac{r}{\rho}$ by formula (21), § 39, Part I., and therefore $\frac{s-\epsilon x}{\log\frac{r}{\rho}}=3C\epsilon=23{\cdot}33$, whence $2C=3{\cdot}022$.

The lines of flow and of equal potential for this case are shown in fig. 6, Plate V., by the part of the figure inside the circle. This figure represents the case of two opposite poles in an unlimited sheet, one of them being twice as strong as the other. One of the flow-lines is then a circle passing through the pole of double strength and having the other pole at its centre. The remaining flow-lines lie half of them within this circle and half of them outside. Consequently, if the sheet be cut along the circumference of this circle (which in the figure is shown as complete), a disk is obtained in which the electrical conditions are the same as in this experiment.

Case 3. Both electrodes in the disk, one on each side the centre and at equal distances from it.

Number of experiments.	r.	AB.	s.	$\frac{s-\epsilon x}{\log\frac{5\,.\,AB}{3\,.\,\rho}}$.
4	13·16	13·16	36·92	16·20
5	7·62	7·62	32·78	16·05
			Mean ……	16·12

Here L becomes $2\log\frac{5\,.\,AB}{3\,.\,\rho}$, whence $\frac{s-\epsilon x}{\log\frac{5\,.\,AB}{3\,.\,\rho}}=2C\epsilon$, and $2C=3{\cdot}132$.

Second set of Resistance experiments, April 1875.

Case 1. Both poles on the edge of the circular disk.

$r=13{\cdot}43$ centims., $\rho={\cdot}163$ centim., $x={\cdot}471$.

Number of experiments.	$2a$.	s.	ϵ.	$R=\frac{s}{\epsilon}-x$.	$\frac{R}{\log\frac{2a}{\rho}}$.
8	26·764	13·03	·9705	12·95	5·85
6	26·773	13·23	·9700	13·07	5·90
				Mean …	5·87

$$L=4\log\frac{2a}{\rho}; \qquad \frac{R}{\log\frac{2a}{\rho}}=4C; \qquad 2C=2{\cdot}935.$$

Case 2. One pole at the centre, the other on the edge (as at a, fig. 4).

$$r=13\cdot43 \text{ centims.}, \quad \rho=\cdot163 \text{ centim.}, \quad x=\cdot425.$$

Number of experiments.	AB $=r$.	s.	ϵ.	$R=\frac{s}{\epsilon}-x$.	$\frac{R}{\log\frac{r}{\rho}}$.
3	13·43	9·37	1·035	8·63	4·504

$$L=3\log\frac{r}{\rho}; \qquad \frac{R}{\log\frac{r}{\rho}}=3C; \qquad 2C=3\cdot003.$$

Case 3. Both poles inside the disk at distances CA, CB respectively from its centre.

$$r=13\cdot43 \text{ centims.}, \quad \rho=\cdot163 \text{ centim.}, \quad x=\cdot38.$$

AB.	CA.	CB.	s.	ϵ.	$R=\frac{s}{\epsilon}-x$.	$\frac{R}{L}$.
13·85	5·55	9·1	7·97	1·0595	7·14	1·63
20·00	10·9	9·1	8·94	1·0395	8·22	1·55
12·20	10·9	4·45	7·64	1·0625	6·81	1·58
7·25	4·5	3·62	6·45	1·0845	5·57	1·63
7·00	7·2	9·0	6·50	1·0850	5·62	1·61
9·00	0·0	9·0	7·17	1·0735	6·30	1·68
12·08	7·7	4·45	7·59	1·0650	6·75	1·64
15·12	7·7	7·7	8·04	1·0615	7·20	1·59
					Mean	...1·615

The flow-lines for this case are indicated in Plate V. fig. 9, where the circle with centre C corresponds with the edge of the disk. In order to get the theoretical resistance for this arrangement, additional poles, of the same signs as A and B respectively, must be assumed at the points A′ and B′; so that, if the conducting sheet were unlimited, the circle with centre C would coincide with a line of flow: that is to say, A′ and B′ must be on the prolongations of the radii through A and B respectively, and at the following distances from the centre, $CA' = \frac{r^2}{CA}$, $CB' = \frac{r^2}{CB}$ (see Part I. §§ 36 and 38). Taking

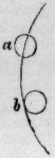
A and B as the middle points of the circles of contact made by the two electrodes, we have (*loc. cit.*)

$$L=\log\frac{(AB)^2 . AB' . A'B}{\rho^2 . AA' . BB'}\,;\quad \frac{R}{L}=C\,;\quad 2C=3{\cdot}23.$$

The values of AB′ and A′B in the above expression were found by laying down on paper, to any convenient scale, the actual positions of the poles from the data AB, CA, CB, CA′, CB′, and then measuring the distances AB′ and A′B. This is a quicker process than the corresponding calculation.

Case 4. One electrode at the centre, the other *touching* the edge (as at *b*, fig. 4).

Fig. 4·



$r=13{\cdot}43$ centims., $\rho={\cdot}163$ centim., $x={\cdot}38$.

Number of experiments.	AB $=r-\rho$.	s.	ϵ.	$R=\frac{s}{\epsilon}-x$.	$\frac{R}{\log\frac{r-\rho}{\sqrt[3]{3}.\rho}}$.
7	13·27	9·13	1·038	8·41	4·80

The poles being assumed to be the centres of the circles of contact made by the electrodes,

$$L \text{ becomes } 3\log\frac{r-\rho}{\sqrt[3]{3}\,.\,\rho}\,;\quad \frac{R}{\log\frac{r-\rho}{\sqrt[3]{3}\,.\,\rho}}=3C\,;\ 2C=3{\cdot}20.$$

Case 5. Both electrodes *touching* the edge.

$r=13{\cdot}43$ centims., $\rho={\cdot}163$ centim., $x={\cdot}38$.

Number of experiments.	AB.	s.	ϵ.	$R=\frac{s}{\epsilon}-x$.	$\frac{R}{\log\frac{AB}{\sqrt{3}\,.\,\rho}}$.
3	24·9	12·42	·983	12·25	6·30
1	16·8	11·58	·996	11·25	6·34
2	8·88	9·985	1·0065	9·54	6·36
2	10·68	10·59	1·016	10·043	6·36
2	22·0	12·34	·988	12·11	6·40
5	26·56	13·35	·969	13·40	6·79
				Mean	... 6·49

In this case, A and B being again taken as the middle points of the circular electrodes,

$$L \text{ becomes } 4\log\frac{AB}{\sqrt{3}\,.\,\rho}; \quad \frac{R}{\log\dfrac{AB}{\sqrt{3}\,.\,\rho}}=4C; \quad 2C=3{\cdot}245.$$

53. *Summary of mean values of* $2C=\dfrac{\log_e 10}{\pi\kappa\delta}$.

Date	1874.		1875.		Mean.
	No. of experiments.	2C.	No. of experiments.	2C.	
Case 1. Both poles on the edge	101	2·943	14	2·935	2·94
Case 2. One in centre and one on the edge	15	3·02	3	3·00	3·01
Case 3. Both inside the disk	9	3·13	10	3·23	3·18
Case 4. One in centre and one touching the edge	......	......	7	3·20	3·20
Case 5. Both touching the edge	......	......	15	3·245	3·245

It will be seen by this Table that the values of C deduced from the several sets of experiments are not quite constant. In explanation of the discrepancies, it is to be observed that the equipotential lines determined by the contact of the electrodes with the tinfoil did not agree quite strictly with equipotential lines due to the arrangement of poles assumed in calculating the values of L, except in the experiments (distinguished as Case 1) where both the electrodes were on the edge of the disk. The differences between the results of these experiments and those with other arrangements of the electrodes are in the direction which would be caused by the actual departure from the theoretical conditions.

The discrepancy is greatest in Case 5, where both the electrodes *touch* the edge of the disk. In this case the additional poles A′ and B′, required in an imaginary unlimited continuation of the conducting sheet in order to make the circumference of the disk a line of flow, would be points outside the circumference, but very near to it and to A and B respectively. If, for an instant, we regard the electrodes as mere points whose distance from the circumference of the disk is very small compared with its radius or with their distance from each other, the equipotential lines for a short distance from A and B will be very nearly indeed a system of lemniscates (see Part I. § 36, at the beginning)—the node of the self-cutting curve, or lemniscate proper, being sensibly on the circumference. The circle of contact formed by each electrode may be regarded as approximately representing one branch of such a lemniscate, if, instead of taking its centre for the actual pole (A or B), we take a point nearer the centre of the disk by $(\sqrt{2}-1)\rho$, where ρ is the radius of the electrode. A recalculation of the experiments of Case 5 in this way reduces the mean value of 2C from 3·245 to 3·11; and a similar correction applied to Case 4 gives $2C=3{\cdot}10$ instead of 3·20. Thus corrected, the results of these experiments agree fairly with those of Case 1.

54. Taking the mean value of 2C obtained with electrodes on the edge of the disk and multiplying it by $\frac{\pi}{\log_e 10}$, we get

4·011,

which ought to be the resistance of a square centimetre of the tinfoil used. For comparison with the value so found, the same quantity was determined by measuring the resistance of an oblong strip of tinfoil, the electrodes being flat pieces of brass, firmly clamped down on the ends of the strip, and making contact with its entire width. Five measurements, made with different lengths of a strip 3 centims. broad, gave as the resistance of a square centimetre

4·032,

the unit being, as before (see § 52), 0·00155 ohm; and five measurements, with a strip 1·5 centim. broad, gave

4·029.

The mean of these determinations is

4·031,

which agrees closely with the value calculated from the experiments with the disks.

55. It seems proper to point out in conclusion, that Professor J. Clerk Maxwell indicated so long ago as 1856 (Brit.-Assoc. Report, 1856, Trans. of Sect. p. 12; see also, for fuller developments, Camb. Phil. Trans. vol. x. p. 27) the fundamental idea of the method of treating questions relating to the flow of electricity in conductors of more than one dimension which was used in the first Part of this paper,—and also to mention that problems presented by finite plane conductors, in cases which become unmanageable by the simple geometrical processes employed by us (as, for instance, the case of rectangular conducting sheets), have been solved by Jochmann (Schlömilch's *Zeitschr. f. Mathemat.* 1865, p. 48) by a method founded on the superposition of the effects of separate poles, and by Heine (Crelle's *Journal*, 1874, vol. lxxix. p. 1) by an application of the geometrical principle of point-point correspondence (*Abbildung*) to Kirchhoff's results.

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PROCEEDINGS

AT THE

MEETINGS OF THE PHYSICAL SOCIETY

OF LONDON.

SESSION 1874–75.

March 21st, 1874.

Prof. Gladstone, F.R.S., President, in the Chair.

Capt. Marshall Hall, Alfred B. Harding, C. T. Woodward, B.Sc., H. W. Eve, C. B. Bourne, M.A., J. Rae, M.D., LL.D., H. Perigal, F.R.A.S., Latimer Clark, C.E., F. Crisp, H. Deacon were admitted Members of the Society.

The following papers were read:—

"On the new Contact Theory of the Galvanic Cell." By J. A. Fleming, B.Sc.

"On the Distribution of a Galvanic Current on entering and leaving a Conducting Medium." By Prof. F. Guthrie, F.R.S.

April 18th, 1874.

Prof. Gladstone, F.R.S., President, in the Chair.

J. H. S. Graham, F.R.G.S., H. Durham, W. Spottiswoode, F.R.S., Dr. H. Müller, F.R.S., W. Garnett, B.A., Rev. J. H. Cooke, F. Clowes, B.Sc., E. Crawshaw, H. E. Armstrong, Ph.D., G. King, C. L. Field, W. H. Pike, Ph.D., A. S. Hobson, S. Lupton, B.A., A. S. L. MacDonald, B.A., R. E. Baynes, B.A., C. A. Kesselmeyer, W. J. Kesselmeyer, J. T. Hurst, T. Jones, F.G.S., A. W. Rücker, M.A., were admitted Members of the Society.

The following papers were read:—

"On Wind Pressures in the Human Chest during performance on Wind Instruments." By Dr. W. H. Stone.

"On the Action of Hydrogen upon finely divided Metals." By A. Tribe, F.C.S.

Mr. F. Clowes, B.Sc., exhibited a simple form of Cell for the Lantern.

May 2nd, 1874.

Prof. Gladstone, F.R.S., President, in the Chair.

W. T. Goolden, B.A., T. Savage, M.A., Dr. Balfour Stewart, F.R.S., Prof. C. Lambert, M.A., T. Threlfall, M.A., A. J. Cownley were admitted Members of the Society.

(Council meeting only.)

May 9th, 1874.

Prof. Gladstone, F.R.S., President, in the Chair.

F. H. Varley, C.E., S. Forsyth, J. R. Adams were admitted Members of the Society.

The following papers were read:—

"On the Saltness or Freshness of the Ice formed on the Sea." By Dr. Rae.

"On the Transportation of Boulders from below Ice to its surface." By Dr. Rae.

"On the Fall in Pitch of Strained Wires through which a Galvanic Current is passing." By Dr. W. H. Stone.

"On a simple means of illustrating Wave Motion." By Mr. W. S. Davis.

May 23rd, 1874.

Prof. Gladstone, F.R.S., President, in the Chair.

F. E. Boughton, E. Rigg, B.A., Rev. A. Freeman, M.A., were admitted Members of the Society.

The following papers were read:—

"On a Modification of the Astatic Galvanometer." By Mr. Ridout.

"On an Absolute Galvanometer." By Prof. F. Guthrie, F.R.S.

June 13th, 1874.

Prof. **Gladstone**, F.R.S., President, in the Chair.

The following papers were read:—

"On an Apparatus for measuring Low Pressure of Gases." By Prof. **M'Leod**.

"On an Arrangement by which the Coloured Rings of Uniaxal and Biaxal Crystals may be shown in a common Microscope fitted with the usual Nicol's Prisms." By Dr. W. H. **Stone**.

Discourse "On the Combination of Colours by Polarized Light." By W. **Spottiswoode**, F.R.S.

June 20th, 1874.

Prof. **Gladstone**, F.R.S., President, in the Chair.

R. J. **Lecky**, F.R.A.S., **Basil Woodd** Smith, F.R.A.S., J. **Stirling**, M.A., H. **Tomlinson**, B.A., R. **Wormell**, M.A., D.Sc., A. C. **Ranyard**, C. W. **Siemens**, F.R.S., T. I. **Pearsall**, S. E. **Phillips** were admitted Members of the Society.

The following paper was read:—

"On Attraction and Repulsion accompanying Radiation." By W. **Crookes**, F.R.S.

Mr. **Woodward** and Prof. **Barrett** exhibited Instruments to illustrate the Interference of Sound.

November 7th, 1874.

Prof. W. G. **Adams**, F.R.S., Vice-President, in the Chair.

The following papers were read:—

"On an Instrument for Multiplying Small Motions." By G. F. **Rodwell**.

"On Graphical Methods of treating certain Elementary Electrical Problems." By Prof. G. C. **Foster**, F.R.S.

"On Salt Solutions and Water of Crystallization." By Prof. F. **Guthrie**, F.R.S.

November 21st, 1874.

Prof. Gladstone, F.R.S., President, in the Chair.

G. Griffiths, M.A., W. Huggins, LL.D., F.R.S., R. W. Okes Voysey, F. D. Brown, J. Baillie Hamilton, R. MacCheane, G. E. Wade, and Albert Macklin were balloted for and elected Members of the Society.

The following papers were read:—

"On a Modified Voltaic Battery." By Prof. M'Leod.

"On the application of Wind to Stringed Instruments." By J. Baillie Hamilton.

December 5th, 1874.

Prof. Gladstone, F.R.S., President, in the Chair.

E. Kinch and J. F. Walker, M.A., were balloted for and elected Members of the Society.

The following papers were read:—

"On a Strophometer." By T. Hearson.

"On an Apparatus for determining the Coefficient of Expansion of a Solid Body." By Prof. G. C. Foster, F.R.S.

January 16th, 1875.

Prof. Gladstone, F.R.S., President, in the Chair.

W. Bottomley, W. H. Perkin, F.R.S., and C. H. Lemann were balloted for and elected Members of the Society.

The following papers were read:—

"On the Electrolysis of some Metallic Chlorides." By Prof. Gladstone, F.R.S., and A. Tribe, F.C.S.

"On the Temperatures of Freezing Mixtures." By Prof. Guthrie, F.R.S.

January 30th, 1875.

Prof. G. C. Foster, F.R.S., Vice-President, in the Chair.

Mostyn J. Salter was balloted for and elected a Member of the Society

The following paper was read:—

"On Electrical Theories." By Dr. A. **Schuster.**

Mr. C. **Becker** exhibited a new Optical Bench.

Annual General Meeting, February 13th, 1875.

Prof. **Gladstone**, F.R.S., President, in the Chair.

The following Report of the Council was read by the President:—

For some years it has been a matter of regret among physicists that there was no Society specially devoted to the reading and discussion of papers connected with their science. The Royal Society readily receives important physical papers; but it is difficult to exhibit experiments, or to discuss them in detail, at Burlington House, and minor or unfinished papers are obviously unsuited for communication to this, the chief of the learned Societies.

A circular which Prof. **Guthrie** addressed in 1873 to the leading physicists elicited a gratifying number of replies, from which it was evident that many were ready to assist in the formation of such a Society, although there were various opinions as to the manner in which it should be constituted. It was, however, decided that it should not differ materially from other learned Societies; and on February 14th, 1874, a General Meeting accepted the Rules drawn up by the Provisional Committee, and requested its Members to act as the Officers and Council for the first year.

The Ordinary Meetings were commenced on the 21st of March under singularly favourable circumstances, as the Lords of the Committee of Council on Education generously placed at the disposal of the Society the Physical Lecture-room and Laboratories at South Kensington, as well as the valuable apparatus they contain. The Society was thus enabled to conduct the Meetings and to provide for the experimental illustration of papers at a very moderate cost, and to offer the advantages of Membership for the small Annual Subscription of £1. The number who have hitherto become Members is 138, of whom 28 have paid the Composition of £10, which entitles them to Life-Membership.

The Society has already to regret the loss of a gifted Member, Mr. W. S. **Davis**, of Derby, who died at the early age of thirty-two years. He was the Head-master of the Derby Central School of

Science; and during the last four years he taught with such skill and care that the study of physical science has been prosecuted with intelligent earnestness in the several classes he established. On the 9th of May he communicated a paper "On a simple means of illustrating wave-motion."

Most of the papers the titles of which are given above have been printed in our Proceedings; and in accordance with an arrangement made with the Editors of the 'Philosophical Magazine,' the original communications have the advantage of immediate publication in that journal.

The first part of the Proceedings was issued in October last, and two or three such parts may be expected annually.

The list shows that our papers have ranged over the sciences of Heat, Light, Sound, Electricity, Molecular and Mechanical Physics; and in the wide field of Physical Science new facts are constantly being discovered and new forms of apparatus devised. There is therefore good ground for hope that the Physical Society will always be abundantly furnished with papers.

The thanks of the Society are specially due to Mr. SPOTTISWOODE, F.R.S., and to Mr. BAILLIE HAMILTON, for discourses which they have delivered during the Session.

The desirability of undertaking the translation of valuable papers published abroad is under the consideration of the Council; and they are in communication with Prof. EVERETT, of Belfast, as to publishing an elaborate paper illustrating the "Centimetre-Gramme-Second" system of units.

The time of meeting has been carefully considered, as the Council were unwilling to add to the already numerous evening engagements of scientific men; and, on the other hand, were reluctant to encroach on the afternoon, which it is usual to keep as a holiday. Saturday afternoon has been chosen, but not without a fear that its selection may prove to be inconvenient to some Members. The Council feel, however, that on the whole the advantages of the time they have adopted outweigh the disadvantages; but they will in this, as in other things, hold themselves ready to carry out the clearly expressed wishes of the majority of the members.

Mr. LATIMER CLARK, in his recent inaugural address to the Society of Telegraphic Engineers, has referred to the suggestion which has been made that there should be an amalgamation of the two Societies, as it is probable that they will often receive papers of common interest. The great difficulty in carrying out the proposed scheme lies in the fact that electricity is only one branch of physics, and that the practical details of telegraphy have little interest for the majority of our Members. On the other hand the Council are not unmindful of the advantages which would attend a close relation with so large and influential a body as the Telegraph Engineers.

The successful formation of our Society is only one indication of the attention which is now devoted to Physics; there is abundant evidence of the rapid rate at which its study is progressing in this country. The last Directory of the Science and Art Department shows that Electricity, Acoustics, Light, and Heat already attract nearly 20,000 students; and the number is increasing annually more rapidly than in the case of any other purely experimental scientific subject.

It is only necessary to add that our Society has been recognized by the Government in the formation of the Committee for carrying out the proposed Loan Exhibition of Scientific Apparatus at South Kensington.

The Society then proceeded to the election of Council and Officers for the ensuing year, and the following gentlemen were declared duly elected:

President.—Prof. J. H. GLADSTONE, F.R.S.

Vice-Presidents.—Prof. W. G. ADAMS, F.R.S.; Prof. G. C. FOSTER, F.R.S.

Secretaries.—Prof. A. W. REINOLD, M.A.; W. CHANDLER ROBERTS.

Treasurer.—Prof. E. ATKINSON.

Demonstrator.—Prof. F. GUTHRIE, F.R.S.

Other Members of Council.—LATIMER CLARK, C.E.; W. CROOKES, F.R.S.; Prof. A. DUPRÉ; Prof. O. HENRICI, F.R.S.; W. HUGGINS, F.R.S.; Prof. H. M'LEOD; W. SPOTTISWOODE, F.R.S.; Dr. H. SPRENGEL; Dr. W. STONE; E. O. WHITEHOUSE.

After the names of the Officers and Council had been announced from the Chair, votes of thanks were proposed to the Lords of the Committee of Council on Education for affording the Society the use of the rooms and apparatus at the Physical Laboratory, South Kensington, to the PRESIDENT, and to Prof. GUTHRIE for his valuable services to the Society.

THE TREASURER IN ACCOUNT WITH THE PHYSICAL SOCIETY, FEBRUARY 13TH, 1875.

Dr.	£	*s.*	*d.*
122 Entrance-Fees in 1874	122	0	0
27 Life Compositions in 1874	270	0	0
95 Annual Subscriptions for 1874	95	0	0
2 Entrance-Fees in 1875	2	0	0
1 Life Composition for 1875	10	0	0
7 Subscriptions for 1875	7	0	0
	£506	0	0

Cr.	£	*s.*	*d.*	£	*s.*	*d.*
Stationery and Books:—						
Weaklin, Flint and Co.	3	9	3			
De La Rue and Co.	2	7	0			
Terry, Stoneman, and Co.	0	10	0			
Cash-Book	0	2	0			
Cheque-Book	0	5	0			
				6	13	3
Printing:—						
Taylor and Francis	28	15	6			
Richardson	3	1	6			
Taylor and Francis	4	3	7			
				36	0	7
Chapman, for attending Meetings in 1874				4	5	0
Postage, Receipt Stamps, Post-office Orders (1874):—						
Secretary	1	9	5			
Treasurer	1	10	6			
				2	19	11
Advertising				2	1	0
Balance in the Bank				454	0	3
				£506	0	0

Examined and found correct.

WILLIAM J. WILSON, } *Auditors.*
HERBERT M'LEOD,

PROCEEDINGS

AT THE

MEETINGS OF THE PHYSICAL SOCIETY

OF LONDON.

SESSION 1875-76.

February 27th, 1875.

Dr. Gladstone, President, and afterwards Mr. Latimer Clark, in the Chair.

The following were elected Members of the Society:—

William Keep; C. G. Morgan.

The following papers were read:—

"On a Method of Projecting the Sodium Spectrum on a Screen." By T. Wills.

"On the Lines of Flow and Equipotential lines in a Uniform Conducting Sheet." By Prof. G. C. Foster and O. J. Lodge.

March 13th, 1875.

Dr. Gladstone, President, in the Chair.

The following were elected Members of the Society:—

Prof. J. D. Everett, D.C.L., F.R.S.E.; C. W. Cooke; T. A. Hearson; Prof. A. B. W. Kennedy.

The following papers were read:—

"On the Electro-Deposition of Iron." By W. C. Roberts.

"On Salt Solutions and Attached Water." By Prof. F. Guthrie.

April 10th, 1875.

Prof. G. C. Foster, Vice-President, in the Chair.

The following was elected a Member of the Society :—

Prof. W. Odling, M.A., F.R.S.

The following paper was read :—

"On the want of Achromatism of the Eye." By Prof. H. M'Leod.

Modified form of Kaleidoscope. By R. Cowper; exhibited by Prof. F. Guthrie.

Modification of Thomson's Quadrant Electrometer; exhibited by W. J. Wilson.

April 24th, 1875.

W. Spottiswoode, Esq., in the Chair.

The following papers were read :—

"On an Auxiliary Air-Pump." By Jerry Barrett.

"On some points connected with Wind Instruments." By Dr. W. H. Stone.

May 8th, 1875.

Dr. Gladstone, President, in the Chair.

The following paper was read :—

Exhibition of Experiments on Attraction and Repulsion accompanying Radiation. By W. Crookes.

"On the Determination of the Velocity of Light." By Prof. Cornu, of the École Polytechnique.

May 22nd, 1875.

Dr. Gladstone, President, in the Chair.

The following papers were read :—

"On a Revolving Polariscope." By W. Spottiswoode.

"On a new form of Polariscope." By Prof. W. G. Adams.

"On Fusion-points and Thermometry." By Dr. Mills.

"On the Electric Conductivity of Anthracite Coal." By H. Bauerman.

"On an Apparatus to illustrate the formation of Volcanic Cones." By Prof. Woodward.

June 12th, 1875.

Dr. GLADSTONE, President, in the Chair.

The following were elected Members of the Society:—

The Lord LINDSAY, M.P.; Prof. J. SYLVESTER, F.R.S.; Sir WILLIAM THOMSON, LL.D., D.C.L., F.R.S.

The following papers were read:—

"On the Electric Conductivity of Glass." By WILDMAN WHITEHOUSE.

"On the Time required for the Double Decomposition of Salts." By the PRESIDENT.

June 26th, 1875.

Prof. G. C. FOSTER, Vice-President, in the Chair.

The following were elected Members of the Society:—

The Earl of ROSSE, D.C.L., F.R.S.; LOUIS SCHWENDLER; R. S. BROUGH.

The following papers were read:—

"On a method of measuring the Electrical Resistance of Liquids." By W. J. WILSON.

"On the Subjective Phenomena of Taste." By Dr. W. H. STONE.

"On a new form of Micrometer for use in Spectrum Analysis." By Dr. W. M. WATTS.

"On the Fundamental Waves in Cylindrical Vessels." By Prof. F. GUTHRIE.

"On a new form of Magneto-Electric Machine." By S. C. TISLEY.

November 13th, 1875.

Dr. GLADSTONE, President, in the Chair.

The following paper was read:—

"On Thermopiles." By Dr. W. H. STONE.

November 27th, 1875.

Prof. G. C. FOSTER, Vice-President, in the Chair.

The following were elected Members of the Society:—

Prof. OSBORNE REYNOLDS, M.A.; Prof. H. J. S. SMITH, M.A., LL.D., F.R.S.; Prof. R. B. CLIFTON, M.A., F.R.S.; C. BUSK; J. M. THOMSON; J. W. W. WAGHORN; W. ESSON, M.A., F.R.S.; F. W. BAYLY; and Prof. R. W. EMERSON MACIVOR.

The following papers were read:—

"Description of Dr. Kerr's Researches on the Depolarizing Action of certain substances when under a powerful Electrical Tension." By Prof. Guthrie.

"On Stationary Liquid Waves." By Prof. Guthrie.

December 11th, 1875.

Dr. Gladstone, President, in the Chair.

The following were elected Members of the Society:—

C. Higgins and S. P. Thompson, B.A., B.Sc.

The following papers were read:—

"On a Simple Chronoscope for Measuring Short Intervals of Time." By O. J. Lodge for Prof. G. C. Foster.

"On an Arrangement for limiting the Charge given to a Leyden Jar." By Prof. M'Leod.

January 15th, 1876.

Dr. Gladstone, President, in the Chair.

The following were elected Members of the Society:—

Sir David Lionel Salomons, Bart.; Arthur R. Granville; and Capt. Abney, R.E.

The following papers were read:—

"On a new Apparatus for exhibiting Wave Motion." By Prof. Woodward.

"On some recent Methods of Spectroscopy." By J. N. Lockyer.

January 29th, 1876.

Dr. Gladstone, President, and afterwards Prof. G. C. Foster, Vice-President, in the Chair.

The following were elected Members of the Society:—

Sir John Conroy, Bart., M.A.; and H. T. Burls.

The following papers were read:—

"On the Polarization of Electrodes in Water free from Air." By J. A. Fleming.

"On the Photography of Fluorescent Substances." By the President.

"On Etheric Force." By S. P. Thompson.

Annual General Meeting, February 12th, 1876.

Dr. **Gladstone**, President, in the Chair.

The following were elected Members of the Society:—

W. R. **Hodgkinson** and H. M. **Hastings**.

The following Report of the Council was read by the President:—

At the last Annual General Meeting the Council was able to draw attention to the success which attended the formation of the Society, and to point to some of the ways for employing its resources with advantage in the future. They have now the satisfaction of stating that the activity with which the Society commenced its work has been well sustained during the past year, as is shown by the list of papers read, many of which have already appeared in the Proceedings.

With reference to communications which are more of the nature of lectures, the thanks of the Society are specially due to M. **Cornu**, of the École Polytechnique—who, on the 8th of May, described and illustrated the methods he had adopted for measuring the velocity of light,—and to Mr. J. **Norman Lockyer**, for his discourse on some recent methods of Spectroscopy, which was delivered on the 15th of January last.

The number of Members has been increased from 138 to 169, of whom 35 have paid the Composition of £10, which entitles them to Life Membership.

The election of many distinguished Physicists during the past year has given the Council much satisfaction, as it affords undoubted evidence of the progress of the Society and of the position it has now attained.

The Society has to regret the loss of two Members, Mr. **Becker** and Mr. **Waugh**.

Carl Ludwig Christian Becker was born at Ratzeburg, in the Grand Duchy of Mecklenburg-Strelitz, on the 16th of July, 1821. He received his general education at the Gymnasium of his birthplace, of which his father was Rector, and studied physical work with **Repsold** at Hamburg, **Kraft** at Vienna, and **Steinheil** at Munich. He came to London in 1849, and joined the firm of **Elliott Brothers** in 1858. He was a Fellow of the Royal Astronomical Society, a Member of the Society of Telegraph Engineers, and an Original Member of our Society. Mr. **Becker** made a communication to the Society of Telegraph Engineers, in January 1875, on the Mance Heliograph; and he read a paper to this Society on a new form of

Optical Bench. He will best be remembered for the beauty and delicacy of the scientific appliances he designed, and for the untiring patience he displayed in constructing instruments to meet the special requirements of other physical workers. There are few of his contemporary Physicists who will not feel that by his death, which occurred on the 3rd of April, 1875, from bronchitis, they have lost a willing helper and a valued friend.

ALEXANDER WAUGH was grandson to the well-known Doctor of Divinity whose name he bore. He was born on the 29th of July, 1836, and educated under Dr. Payne, late President of the College of Preceptors. From an early age he showed great love for physical science, and also for music—two tastes not uncommonly united.

A course of lectures by FARADAY in 1851 gave additional impulse to his studies in a scientific direction. He received from the great Physicist that kind attention and encouragement which were always granted by him to intelligent students. In 1855 WAUGH entered at University College, and a few years later joined his father in an extensive Pharmaceutical business.

His favourite subject was Optics, especially spectroscopic research and the laws of Polarized Light. A paper by him "On the Spectrum of Polarized Light," was read before the Chemical Society, and also brought under the notice of the British Association by Dr. GLADSTONE. In this communication, as early as 1857, he described the beautiful absorption-bands to be seen traversing the spectrum of a polarized ray transmitted through a doubly refracting film. He constructed an apparatus for their demonstration and measurement, thus in a measure anticipating the later researches of SPOTTISWOODE and SORBY. He also constructed with his own hands one of the finest spectroscopes yet made, in the manipulation of which he was singularly dexterous.

The sudden death by drowning of his only brother gave his health, always delicate, a severe shock; but, though ordered perfect repose, he was unable to tear himself from his favourite studies. On the 12th of October, 1875, an epileptiform seizure, indicating that the brain was physically unequal to its mental activity, terminated in death before his fortieth year. He was, however, no unworthy pupil of his great master FARADAY.

In the last Report it was stated that the Council was in communication with Prof. EVERETT, of Belfast, as to the publication of an elaborate paper illustrating the Centimetre-Gramme-Second System of Units. This work, which was issued to Members in August last, is based on the recommendations of a Committee of the British Association, and consists of a collection of physical data concisely presented on the above system, a complete account being added of the Theory of Units.

The Council is now in communication with the family of the late Sir CHARLES WHEATSTONE, with a view to the publication of his papers.

At the last General Meeting the number of Members of Council was increased from 13 to 17, and an additional Secretary was appointed—changes which have materially conduced to the efficient conduct of the Society's business.

During the last Session full abstracts of the papers and discussions have been furnished to the Scientific Journals by one of the Secretaries, who still continues to edit these reports, the preparation of which has been intrusted to a Member of the Society, whose services the Council has had the good fortune to secure.

A Committee of Council was appointed to revise the Bye-Laws, and it has proposed the changes which will be submitted to the Meeting today. This was considered necessary with a view to the formal registration of the Society in accordance with the provisions of the 23rd Section of the Companies' Act (1862) Amendment, 30 and 31 of Vic. Cap. 131, a—step which should not longer be delayed.

The Council is fully sensible of the advantage the Society derives from the use of the Physical Lecture-Room and Laboratories which have been so generously placed at its disposal by the Lords of the Committee of Council on Education, to whom the best thanks of the Members are due.

Friendly communications with other Physicists are of course most desirable; and the Society has already effected an interchange of publications with the Imperial University of Moscow and the Physical Societies of Paris and Berlin. The question of electing Foreign Members will receive due attention during the forthcoming Session.

The question of creating Associates of the Society has been discussed; but it was considered preferable that the Council should seek power to grant admission, to all Meetings of the Society, to persons who are not Members.

The Council desires, in concluding this Report, to record its gratitude for the important services which Dr. GUTHRIE has rendered to the Society. The Members will bear in mind that the office of Demonstrator was formerly an important one in the Royal Society; and the Council believes that much might be gained if arrangements could be made for reproducing, before this Society, the experiments described in original papers which appear from time to time in this country and abroad.

Certain changes in the Bye-Laws were discussed and agreed to.

Messrs. A. S. Hobson and W. H. Walenn having been appointed Scrutators,

The Society proceeded to the election of the Council and Officers for the ensuing year; and the following gentlemen were declared duly elected:

President.—Prof. G. C. Foster, F.R.S.

Vice-Presidents.—Prof. W. G. Adams, F.R.S.; W. Spottiswoode, LL.D., F.R.S.

Secretaries.—Prof. A. W. Reinold, M.A.; W. C. Roberts, F.R.S.

Treasurer.—Prof. E. Atkinson.

Demonstrator.—Dr. F. Guthrie, F.R.S.

Other Members of Council.—Latimer Clark, C.E.; Prof. A. Dupré, F.R.S.; W. Huggins, D.C.L., F.R.S.; Prof. H. M'Leod; Dr. C. W. Siemens, D.C.L., F.R.S.; Dr. H. Sprengel; Dr. W. H. Stone; Sir William Thomson, LL.D., D.C.L., F.R.S.; Prof. W. C. Unwin, B.Sc.; E. O. W. Whitehouse.

After the names of the Council and Officers had been announced from the Chair, votes of thanks were passed:—to the Lords of the Committee of Council on Education for affording the Society the use of the rooms and apparatus at the Physical Laboratory, South Kensington; to the President; to Prof. Guthrie for his valuable services to the Society; and to the Treasurer and Secretaries.